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Analysis of Characteristics and Parameter Estimation of the Birnbaum-Saunders Distribution on Daily Gold Price

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ABSTRACT

This study aims to analyze the distribution characteristics of daily gold price data from February 2 to June 1, 2026, using the Birnbaum-Saunders distribution. The parameter estimation method utilized is the Maximum Likelihood Estimation (MLE) with numerical solutions using the Quasi-Newton (BFGS) method, given that the resulting system of equations is non-linear and lacks a closed-form analytical solution. Based on the estimation results, the shape parameter (α) is 0.0400 and the scale parameter (β) is 2,632,125.38. The goodness-of-fit testing was conducted using the Kolmogorov-Smirnov (K-S) and Anderson-Darling (A-D) tests. The results indicate that the null hypothesis (H_0) is not rejected at a 0.05 significance level, suggesting that the Birnbaum-Saunders distribution is appropriate for modelling the daily gold price data. Furthermore, PDF and CDF visualizations demonstrate a high degree of conformity between the theoretical model and the empirical data, indicating that the Birnbaum-Saunders distribution is a suitable model for describing the distribution of daily gold prices in this period. These results rely on the assumption that daily prices are independent, which is a simplification because daily gold prices are serially dependent. A comparison with the Normal, Lognormal, Gamma, and Weibull distributions based on AIC and BIC showed that the Birnbaum-Saunders distribution was comparable to the Normal, Lognormal, and Gamma distributions (AIC differences below 0.25) and clearly better than the Weibull distribution.

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1. INTRODUCTION

Gold has long been recognised as one of the most stable investment and hedging instruments amid global economic uncertainty. Gold price fluctuations are driven by various macroeconomic factors, such as inflation, exchange rates, geopolitical conditions and global economic instability. Under economic instability, demand for gold tends to rise because gold is considered relatively safe compared with other investments (Baur & McDermott, 2010). A good understanding of the distributional characteristics of gold prices is therefore essential for investors, market participants and policymakers in making data-driven decisions.

In financial data analysis, choosing an appropriate distribution model is crucial. Commodity prices such as gold often show asymmetry (skewness) and tail shapes that differ from the normal distribution, so the normal distribution is not always suitable for modelling them (Balakrishnan & Kundu, 2019). More flexible alternative distributions are therefore needed.

One suitable distribution is the Birnbaum-Saunders distribution, first introduced by Z. W. Birnbaum and S. C. Saunders to model the failure times of a component (Birnbaum & Saunders, 1969). This distribution has been reported to handle positively skewed data with asymmetric variance effectively and has been used in several fields, including finance and economics (Martínez-Flórez et al., 2019; Bhatti, 2010). In Indonesia, its application to gold price data is still limited, so this study is needed as a preliminary investigation.

Previous studies show that the Birnbaum-Saunders distribution has been developed and widely applied, from modelling the fatigue life of materials to financial data (Balakrishnan & Kundu, 2019; Bhatti, 2010; Martínez-Flórez et al., 2019). In Indonesia, Langowuyo et al. (2025) applied this distribution to data on the average years of schooling in Merauke Regency. For financial data, the Birnbaum-Saunders distribution and its variants have been used, among others, to model right-skewed financial data and related risk measures such as Value at Risk (Naderi et al., 2020) and to model the durations between high-frequency stock transactions (Saulo et al., 2019). However, to the best of the authors' knowledge, its application to daily gold prices is still rare. It should be noted that daily gold prices are time-series data, so the price on a given day may depend on the price on the previous day. In this study, daily prices are treated as independent observations in order to describe the price distribution over the observation period; this assumption is stated in Section 2.4, and its consequences are discussed in the limitations and suggestions. Studies on the distribution of gold prices have so far focused mainly on returns. Shen et al. (2017) found that the skewed t distribution best fits daily gold spot returns, while Chinhamu et al. (2015, 2017) showed that heavy-tailed distributions such as the generalised Pareto outperform the normal distribution in estimating Value at Risk in the gold market. In Indonesia, Antam gold returns have also been reported to be non-normal and heavy-tailed (Sulistiowati et al., 2021; Kamalia & Budiyanto, 2026). Unlike these studies, the present study models the distribution of positive-valued gold price levels, so a distribution with positive support such as the Birnbaum-Saunders is relevant. Based on this review, the novelty of this study lies in applying the Birnbaum-Saunders distribution to daily gold prices for February–June 2026, together with a comparison with alternative distributions and a practical interpretation of the parameters.

In distribution analysis, parameter estimation is an important step. One of the most commonly used estimation methods is Maximum Likelihood Estimation (MLE), because it produces efficient and consistent estimators. However, for the Birnbaum-Saunders distribution the likelihood equations are non-linear and generally do not have a closed-form analytical solution. Numerical optimisation methods such as the Quasi-Newton method, in

particular the BFGS algorithm, are therefore needed to obtain the parameter estimates (Ng et al., 2003; Jiang et al., 2017).

Based on the above, this study aims to analyse the distributional characteristics of daily gold prices from 2 February to 1 June 2026 using the Birnbaum-Saunders distribution. The study focuses on estimating the distribution parameters by Maximum Likelihood Estimation (MLE) with the Quasi-Newton (BFGS) algorithm, evaluating the model fit with the Kolmogorov-Smirnov and Anderson-Darling goodness-of-fit tests, and comparing the model with alternative distributions (Normal, Lognormal, Gamma and Weibull) using the AIC and BIC criteria.

2. METHOD

This study uses a quantitative approach with statistical analysis to model daily gold prices from 2 February to 1 June 2026 using the Birnbaum-Saunders distribution. The approach aims to analyse the distributional characteristics of the data, estimate the distribution parameters and evaluate the fit of the distribution model to the observed data using goodness-of-fit tests.

2.1 Data Source

The data used in this study are daily gold prices from 2 February to 1 June 2026, with 86 observations. The data are continuous quantitative data, and all observed values are positive. This matches the support of the Birnbaum-Saunders distribution, namely $x > 0$.

The data were obtained from Investing.com (XAU/IDR Historical Data), accessed on 13 June 2026, in rupiah per gram (XAU/IDR closing prices per troy ounce converted to grams, with 1 troy ounce = 31.1035 grams). The 86 observations cover trading days only, so there are fewer observations than calendar days in the period. The data are publicly available secondary data and contain no personal information. The study does not involve human subjects, so no informed consent was required, and the data were used for academic purposes only. Prices per gram were rounded to the nearest rupiah before analysis.

Daily gold prices were chosen because they fluctuate under the influence of various economic factors, such as inflation, exchange rates, geopolitical conditions and global market uncertainty. A probability distribution analysis is therefore needed to understand the distribution of the data mathematically.

2.2 Birnbaum-Saunders Distribution

The Birnbaum-Saunders distribution is a continuous distribution first introduced by Z. W. Birnbaum and S. C. Saunders to model fatigue-life data (Birnbaum & Saunders, 1969). It is well suited to continuous data with positive skewness.

A continuous random variable X is said to follow the Birnbaum-Saunders distribution with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$, written as:

$$\boldsymbol{X \sim BS(\alpha, \beta)}$$

and its probability density function (PDF) is defined as:

$$f(x;\alpha,\beta) = \frac{1}{2\alpha\beta\sqrt{2\pi}}\left[\left(\frac{\beta}{x}\right)^{1/2} + \left(\frac{\beta}{x}\right)^{3/2}\right] exp\left[-\frac{1}{2\alpha^2}\left(\frac{x}{\beta} + \frac{\beta}{x} - 2\right)\right]$$

for $x > 0$ with $\alpha > 0$ and $\beta > 0$.

The cumulative distribution function (CDF) of the Birnbaum-Saunders distribution is given by (Balakrishnan et al., 2009):

$$F(x) = \Phi\left[\frac{1}{\alpha}\left(\sqrt{\frac{x}{\beta}} - \sqrt{\frac{\beta}{x}}\right)\right]$$

where $\Phi(.)$ is the cumulative distribution function of the standard normal distribution.
An important property of the Birnbaum-Saunders distribution is that the parameter β is the median of the distribution ($F(\beta)$=0.5), while the parameter α controls its shape; the smaller the value of α, the more symmetric the distribution and the closer it is to the normal distribution (Balakrishnan & Kundu, 2019).

2.3 Parameter Estimation

Parameter estimation aims to find the best approximate values of the shape parameter (α) and the scale parameter (β) from the gold price sample. The method used in this study is Maximum Likelihood Estimation (MLE), which finds the parameter values that maximise the probability function of the observed data (Casella & Berger, 2024). The MLE procedure consists of the following steps:

a. Specify the probability density function (PDF)
b. Form the likelihood function
   The likelihood function is the product of the PDF over all observations:
$$L(\alpha, \beta) = \prod_{i=1}^{n} f(x_i; \alpha, \beta)$$
c. Form the log-likelihood function
   To turn the product into a sum, the likelihood function is transformed using the natural logarithm:
$$\ell(\alpha, \beta) = \ln L(\alpha, \beta) = \sum_{i=1}^{n} \ln f(x_i; \alpha, \beta)$$
d. Compute the first partial derivatives with respect to the parameters
   The log-likelihood function is then differentiated partially with respect to the shape parameter (α) and the scale parameter (β).
   First derivative with respect to the shape parameter (α):
$$\frac{\partial \ell}{\partial \alpha}$$
   First derivative with respect to the scale parameter (β):
$$\frac{\partial \ell}{\partial \beta}$$
e. Set the first derivatives equal to zero
   First derivative with respect to the shape parameter (α):
$$\frac{\partial \ell}{\partial \alpha} = 0$$
   First derivative with respect to the scale parameter (β):
$$\frac{\partial \ell}{\partial \beta} = 0$$
f. Find the maximum likelihood estimates with the Quasi-Newton (BFGS) method
   Because the system of first-derivative equations is non-linear and has no closed-form analytical solution, the parameters are estimated with an iterative numerical method (Teimouri et al., 2013). This study uses the Quasi-Newton (BFGS) method to obtain

convergent maximum likelihood estimates $\hat{\alpha}$ and $\hat{\beta}$ (Jiang et al., 2017). The optimisation uses the median of the observed data as the starting value for the scale parameter (β). The starting value of the shape parameter was set to $\alpha_o = 0.0400$ (the modified moment estimator $\alpha_o = \{2[(s/r)^{½} - 1]\}^{½}$, where s is the arithmetic mean and r the harmonic mean of the data; Ng et al., 2003), and the starting value of the scale parameter, β_o, was the sample median, Rp2,614,416.50. The iterations stopped when the gradient norm of the log-likelihood fell below 10^{-3}, and the optimisation converged after 6 iterations. For numerical stability, the data were divided by their median before optimisation, and $\hat{\beta}$ was then transformed back to the original scale.

2.4 Goodness-of-Fit Test

The goodness-of-fit test is used to verify empirically whether the fitted Birnbaum-Saunders model can represent the distribution of daily gold prices. The hypotheses tested are:

H_0 : *Daily gold prices follow the Birnbaum-Saunders distribution*

H_1 *: Daily gold prices do not follow the Birnbaum-Saunders distribution*

Assumption used. MLE and the Kolmogorov-Smirnov and Anderson-Darling tests assume that the observations $x_1, x_2, \ldots, x_n$ form a random sample of independent and identically distributed (iid) variables. In this study, daily gold prices are therefore assumed to be independent across days, that is, the price on a given day is not affected by the price on the previous day. This assumption is a simplification, because daily gold prices may be autocorrelated; the goodness-of-fit results are therefore interpreted as a description of the (marginal) distribution of gold prices over the observation period, not as a model of price dynamics over time.

Following Razali & Wah (2011), to obtain comprehensive and valid conclusions, the model fit is evaluated with two non-parametric tests simultaneously:

a. Kolmogorov-Smirnov (K-S) test

The K-S test measures the maximum deviation between the empirical and theoretical distributions (Massey, 1951).

$$D_n = \sup_x |F_n(x) - F_0(x)|$$

where $F_n(x)$ is the empirical distribution function (the proportion of data values ≤ x), $F_0(x)$ is the theoretical Birnbaum-Saunders CDF with the estimated parameters, and sup denotes the largest value (supremum) of the absolute difference between the two functions over all x.

This test is more sensitive in the centre of the distribution. The decision rule is to accept H_0 if the p-value >0.05 at the 95% confidence level.

b. Anderson-Darling (A-D) test

The Anderson-Darling test gives more weight to the tails of the distribution (Anderson & Darling, 1952).

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\left[\ln F_0\left(x_{(i)}\right) + \ln\left(1 - F_0\left(x_{(n+1-i)}\right)\right)\right]$$

where $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$ are the data sorted from smallest to largest and F_0 is the theoretical Birnbaum-Saunders CDF. A small value of A^2 indicates a good fit between the data and the model.

This test is more sensitive to departures in the extreme values.

2.5 Data Analysis Steps

All statistical analyses and computations in this study were carried out in Python on the Google Colab platform. The data analysis steps are as follows:

a. Data collection: entering the 86 daily gold price observations from 2 February to 1 June 2026.
b. Descriptive statistics: computing summary statistics of the raw data to detect the shape of the distribution (mean, median, standard deviation, range, skewness, kurtosis).
c. MLE parameter estimation: evaluating the log-likelihood function on the data and running the Quasi-Newton (BFGS) algorithm to obtain the estimates $\hat{\alpha}$ and $\hat{\beta}$, namely the shape parameter $\hat{\alpha}$, which describes the skewness and relative variation of the price distribution (for small α, α approximates the coefficient of variation), and the scale parameter $\hat{\beta}$, which is the median of the price distribution, for the sample of n = 86 daily gold prices. The starting values are α_0 from the modified moment estimator and β_0 = the sample median.
d. Hypothesis testing: computing the Kolmogorov-Smirnov test statistic (D and p-value) and the Anderson-Darling test statistic.
e. Visualisation: plotting the fitted probability density (PDF) and cumulative distribution (CDF) curves against the data to compare the theoretical shape with the observed data. The visual evaluation was carried out in two ways: (i) comparing the density histogram of the data with the theoretical PDF to assess the agreement in location, spread and skewness; and (ii) comparing the empirical distribution function (ECDF) with the theoretical CDF, where the largest vertical distance between the two curves equals the K-S test statistic.

2.6 Goodness-of-Fit Correction and Model Comparison

Because α and β are estimated from the same data that are tested, the K-S and A-D statistics no longer follow their standard distributions, so the standard p-values tend to be too large (Lilliefors, 1967). Therefore, in addition to the standard p-values, a parametric bootstrap was carried out: B = 2000 samples were generated from the fitted distribution, the parameters were re-estimated for each bootstrap sample, and the p-value was computed as the proportion of bootstrap statistics greater than or equal to the statistic of the original data.

To assess whether the Birnbaum-Saunders distribution is better than the alternatives, it was compared with the Normal, Lognormal, Gamma and Weibull distributions (all with two parameters) using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) (Akaike, 1974; Schwarz, 1978), namely AIC = $2k - 2\ell$ and BIC = $k \ln(n) - 2\ell$, where k is the number of parameters, n the number of observations and ℓ the maximised log-likelihood. A larger maximised log-likelihood ℓ (less negative) indicates a better fit; because AIC and BIC contain the term -2ℓ, the model with smaller AIC and BIC values is the better model. The bootstrap K-S and A-D tests were also computed for each distribution.

3. RESULTS AND DISCUSSION

3.1 Descriptive Statistics

Descriptive statistics were computed to give an overview of the distribution of daily gold prices from 2 February to 1 June 2026. The results are summarized in Table 1.

Table 1. Descriptive Statistics of Gold Prices (2 February – 1 June 2026)

No	Statistic	Value
1	Number of observations (n)	86
2	Minimum	2,380,281
3	Maximum	2,884,960
4	Mean	2,634,231.30
5	Median	2,614,416.50
6	Mode	None (all values are distinct)
7	Variance	11,242,373,559.93
8	Standard deviation	106.030.06
9	Skewness	0.12
10	Kurtosis	-0.33

Note: prices are in rupiah per gram (Rp/gram); variance in (Rp/gram)²; skewness and kurtosis are unitless.

Table 1 shows that the mean gold price over the period is Rp2,634,231.30. The *skewness* of 0.12 indicates a very slight right skew (*positively skewed*), meaning that the data are nearly symmetric with slightly more observations at lower values. The *kurtosis* of −0.33 indicates a *platykurtic* distribution, which is flatter than the normal distribution. Given this pattern, the Birnbaum-Saunders distribution is considered worth examining as a model for the distribution of these gold prices.

3.2 MLE Parameter Estimation

a. Specify the probability density function (PDF):

For a random variable $x > 0$ with shape parameter $\alpha > 0$ and scale parameter $\beta > 0$, the probability density function is:

$$f(x;\alpha,\beta) = \frac{1}{2\alpha\beta\sqrt{2\pi}}\left[\left(\frac{\beta}{x}\right)^{1/2} + \left(\frac{\beta}{x}\right)^{3/2}\right] exp\left[-\frac{1}{2\alpha^2}\left(\frac{x}{\beta} + \frac{\beta}{x} - 2\right)\right]$$

b. Form the likelihood function

The *likelihood* function $L(\alpha,\beta)$ represents the joint probability function of the independent sample. This product form holds only if the observations are independent, in line with the assumption of independent daily prices in Section 2.4. For observations $x_1, x_2, \ldots, x_n$ following the Birnbaum-Saunders distribution, the *likelihood* function is defined as the product of the probability density functions (PDF) of all observations:

$$L(\alpha,\beta) = \prod_{i=1}^{n} f\,(x_i;\alpha,\beta)$$

Substituting the Birnbaum-Saunders PDF defined above gives:

$$L(\alpha,\beta) = \prod_{i=1}^{n}\left[\frac{1}{2\alpha\beta\sqrt{2\pi}}\left(\left(\frac{\beta}{x_i}\right)^{1/2} + \left(\frac{\beta}{x_i}\right)^{3/2}\right)\exp\left(-\frac{1}{2\alpha^2}\left(\frac{x_i}{\beta} + \frac{\beta}{x_i} - 2\right)\right)\right]$$

c. Form the log-likelihood function

To simplify the estimation, the *likelihood* function $L(\alpha,\beta)$ is transformed into the *log-likelihood* function $\ell(\alpha,\beta)$ by taking the natural logarithm $(\ln)$. This transformation does not change the location of the maximum, but it simplifies the partial derivatives because the logarithm turns products into sums.

$$\ell(\alpha,\beta) = \ln L\,(\alpha,\beta) = \sum_{i=1}^{n} \ln f\,(x_i;\alpha,\beta)$$

Substituting the Birnbaum-Saunders PDF and simplifying gives the following *log-likelihood* function:

$$\ell(\alpha,\beta) = -n\ln\left(2\alpha\beta\sqrt{2\pi}\right) + \sum_{i=1}^{n}\ln\left[\left(\frac{\beta}{x_i}\right)^{1/2} + \left(\frac{\beta}{x_i}\right)^{3/2}\right] - \sum_{i=1}^{n}\frac{1}{2\alpha^2}\left[\frac{x_i}{\beta} + \frac{\beta}{x_i} - 2\right]$$

d. Compute the first partial derivatives with respect to the parameters

To obtain the maximum likelihood estimators $(\hat{\alpha}$ and $\hat{\beta})$, we compute the first partial derivatives of $\ell(\alpha,\beta)$ with respect to each parameter.

1. Partial derivative with respect to the shape parameter $(\boldsymbol{\alpha})$:

$$\frac{\partial\ell}{\partial\alpha} = -\frac{n}{\alpha} + \frac{1}{\alpha^3}\sum_{i=1}^{n}\left[\frac{x_i}{\beta} + \frac{\beta}{x_i} - 2\right] = 0$$

From the equation above, $\hat{\alpha}^2$ can be expressed as a function of $\hat{\beta}$:

$$\hat{\alpha}^2 = \frac{1}{n}\sum_{i=1}^{n}\left[\frac{x_i}{\hat{\beta}} + \frac{\hat{\beta}}{x_i} - 2\right]$$

2. Partial derivative with respect to the scale parameter $(\boldsymbol{\beta})$:

$$\frac{\partial\ell}{\partial\beta} = -\frac{n}{\beta} + \sum_{i=1}^{n}\frac{\frac{1}{2\beta}\left(\frac{\beta}{x_i}\right)^{1/2} + \frac{3}{2\beta}\left(\frac{\beta}{x_i}\right)^{3/2}}{\left(\frac{\beta}{x_i}\right)^{1/2} + \left(\frac{\beta}{x_i}\right)^{3/2}} - \frac{1}{2\alpha^2}\sum_{i=1}^{n}\left[\frac{1}{x_i} - \frac{x_i}{\beta^2}\right]$$

e. Set the first derivatives equal to zero

Following the principle of *Maximum Likelihood Estimation*, the estimates that maximise the *log-likelihood* function are obtained where its gradient is zero. Setting the partial derivatives in step (d) equal to zero gives the following system of non-linear equations:

1) Equation for the shape parameter (α):

$$-\frac{n}{\alpha} + \frac{1}{\alpha^3}\sum_{i=1}^{n}\left[\frac{x_i}{\beta} + \frac{\beta}{x_i} - 2\right] = 0$$

2) Equation for the scale parameter (β):

$$-\frac{n}{\beta} + \sum_{i=1}^{n}\frac{\frac{1}{2\beta}\left(\frac{\beta}{x_i}\right)^{1/2} + \frac{3}{2\beta}\left(\frac{\beta}{x_i}\right)^{3/2}}{\left(\frac{\beta}{x_i}\right)^{1/2} + \left(\frac{\beta}{x_i}\right)^{3/2}} - \frac{1}{2\alpha^2}\sum_{i=1}^{n}\left[\frac{1}{x_i} - \frac{x_i}{\beta^2}\right] = 0$$

f. Find the maximum likelihood estimates with the Quasi-Newton method

Because the system of first-derivative equations (step e) is non-linear and has no closed-form analytical solution, the parameters are estimated with an iterative numerical method. This study uses the Quasi-Newton (BFGS**)** method to obtain convergent maximum likelihood estimates $\hat{\alpha}$ and $\hat{\beta}$.

The optimisation uses the median of the observed data as the starting value for the scale parameter $(\hat{\beta})$. The iterations give the following estimates:

- Shape parameter estimate $\hat{\alpha}$=0.0400
- Scale parameter estimate $\hat{\beta}$ = 2,632,125.38

3.3 Goodness-of-Fit Test

To test whether daily gold prices follow the Birnbaum-Saunders distribution, the following hypotheses are tested:

H_0: The data follow the Birnbaum-Saunders distribution.

H_1: The data do not follow the Birnbaum-Saunders distribution.

a) *Kolmogorov-Smirnov* (K-S) test

The Kolmogorov-Smirnov test gives a statistic of 0.0775 with a *p-value* of 0.6511. Because the *p-value* > 0.05, H_0 is not rejected. This indicates that there is no significant difference between the empirical distribution of gold prices and the continuous Birnbaum-Saunders model. With the parametric bootstrap correction (B = 2000), the K-S p-value is 0.2424.

b) Anderson-Darling (A-D) test

The Anderson-Darling test gives a statistic A^2 of 0.6851. This relatively small value further supports that the Birnbaum-Saunders model represents the gold price data reasonably well, including in the tails. With the parametric bootstrap correction, the A-D p-value is 0.0720.

Based on the Kolmogorov-Smirnov and Anderson-Darling tests, the null hypothesis (H_0) is not rejected at the 0.05 significance level. This indicates that the Birnbaum-Saunders distribution with parameters $\hat{\alpha}$=0.0400 and $\hat{\beta}$ = 2,632,125.38 is suitable for modelling daily gold prices from 2 February to 1 June 2026. The *p-values* above 0.05 and the small Anderson-Darling statistic show that there is insufficient evidence to reject the fit of the Birnbaum-Saunders distribution, both in the centre and in the extremes. However, these results should be interpreted with caution because, with 86 observations, the power of the tests is limited; the K-S and A-D tests assume independent daily prices, whereas daily prices are likely to depend on the previous day; and passing a goodness-of-fit test does not mean that the model is better than other distributions (see Section 3.5).

3.4 Visualisation of the PDF and CDF

This section compares the empirical daily gold prices with the theoretical probability density function (*Probability Density Function* - PDF) and cumulative distribution function (*Cumulative Distribution Function* - CDF) of the Birnbaum-Saunders distribution with the estimated parameters $\hat{\alpha}$=0.0400 and $\hat{\beta}$ = 2,632,125.38.

a) Probability Density Function (PDF) Plot

The PDF plot in Figure 1(a) compares the frequency distribution of the empirical data with the theoretical Birnbaum-Saunders curve. The histogram shows the data concentrated around the centre of the gold prices, while the theoretical curve follows the overall shape of the data smoothly and in line with the empirical pattern.

b) Cumulative Distribution Function (CDF) Plot

The CDF plot in Figure 1(b) compares the empirical distribution function with the smooth theoretical distribution function. The theoretical CDF agrees closely with the empirical distribution of gold prices; the maximum vertical distance between the two curves is the Kolmogorov-Smirnov statistic (D_n=0.0775). This value quantifies the good agreement between the continuous model and the observed gold prices.

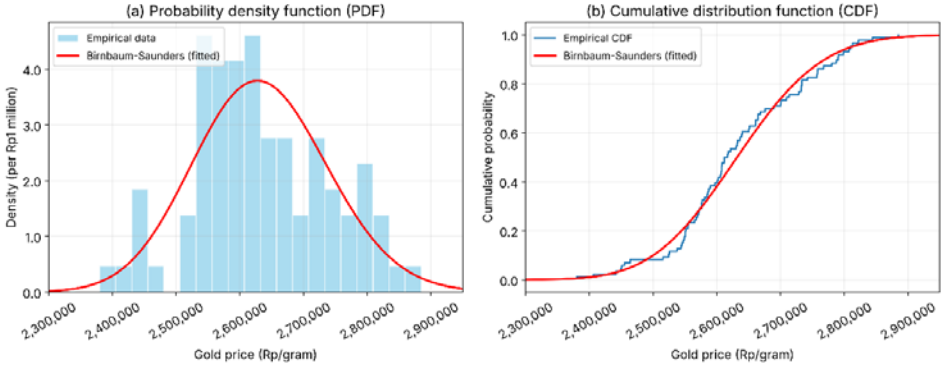

Figure 1. *Empirical versus Theoretical Birnbaum-Saunders PDF and CDF.*

Visually, the Birnbaum-Saunders distribution captures the centre and spread of daily gold prices well. The agreement in the CDF plot, where the model follows the empirical curve with minimal deviation, explains why the statistical tests (K-S and A-D) fail to reject the null hypothesis. This confirms that the Birnbaum-Saunders distribution is a suitable model for the distribution of daily gold prices in this period (Langowuyo et al., 2025; Martínez-Flórez et al., 2019).

3.5 Comparison with Alternative Distributions

To assess whether the Birnbaum-Saunders distribution fits better than other distributions, Table 2 compares the five distributions by log-likelihood (log-L), AIC, BIC, and the parametric bootstrap p-values of the K-S and A-D tests.

Table 2. Comparison of the Birnbaum-Saunders Distribution with Alternative Distributions

Distribution	log-L	AIC	BIC	p-value K-S	p-value A-D
Birnbaum-Saunders	-1.116.56	2.237.11	2.242.02	0.2424	0.0720
Normal	-1.116.67	2.237.35	2.242.25	0.1414	0.0585
Lognormal	-1.116.56	2.237.12	2.242.03	0.2384	0.0705
Gamma	-1.116.58	2.237.15	2.242.06	0.1799	0.0630
Weibull	-1.122.11	2.248.22	2.253.13	0.0005	0.0010

The Birnbaum-Saunders distribution has the largest log-likelihood (−1,116.56, almost equal to the Lognormal) and therefore the smallest AIC and BIC (2,237.11 and 2,242.02), but its differences from the Normal (ΔAIC = 0.23), Lognormal (0.00) and Gamma (0.04) distributions are less than 2 (computed from unrounded AIC values), so the four distributions are considered equivalent. The Weibull distribution is clearly worse (ΔAIC = 11.11) and is rejected at the 0.05 level by both bootstrap tests (p = 0.0005 and 0.0010). In the bootstrap A-D test, the p-value of the Birnbaum-Saunders distribution (0.0720) is above 0.05 but not by much, so the fit in the tails should be read with caution. Note that with a small $\hat{\alpha} = 0.0400$ and a skewness of only 0.12, the Birnbaum-Saunders distribution is close to symmetric, so its results can be very similar to those of the Normal distribution. When the differences in information criteria are small, the Birnbaum-Saunders distribution is considered as adequate as these alternatives; its added value lies in its positive-only support and easily interpretable parameters (Balakrishnan & Kundu, 2019).

3.6 Parameter Interpretation and Practical Implications

The scale parameter $\hat{\beta}$ = Rp2,632,125.38 is the median of the model and is close to the sample median of Rp2,614,416.50. The shape parameter $\hat{\alpha} = 0.0400$ is very small; for small α, α approximately acts as the coefficient of variation, that is, the standard deviation relative to the mean. This is consistent with the sample coefficient of variation of 4.03% (106,030.06 / 2,634,231.30), so gold price fluctuations in this period are relatively small, about 4% of the mean price. The model mean $\hat{\beta}(1 + \hat{\alpha}^2/2)$ = Rp2,634,231.29 and the model standard deviation of Rp105,395.38 are also very close to the sample mean and standard deviation in Table 1.

Price quantiles can be computed from the fitted distribution, $x_q = \frac{\hat{\beta}}{4}\left[\hat{\alpha} z_q + \sqrt{\left(\hat{\alpha} z_q\right)^2 + 4}\right]^2$, where z_q is the standard normal quantile (Table 3). The model predicts that 90% of daily prices lie between Rp2,464,542.45 and Rp2,811,103.55 (P5–P95), and only 1% of daily prices fall below Rp2,398,315.85. Such a lower quantile can serve market participants as a simple benchmark for a low-price level (analogous to an unconditional Value at Risk), but it is not a price forecast because the model ignores dependence over time.

Table 3. Gold Price Quantiles under the Birnbaum-Saunders Model ($\hat{\alpha}$ = 0.0400; $\hat{\beta}$ = 2,632,125.38)

Quantile	Cumulative probability	Model price (Rp)
P1	0.01	2,398,315.85
P5	0.05	2,464,542.45
P50 (median)	0.50	2,632,125.38
P95	0.95	2,811,103.55
P99	0.99	2,888,728.79

The limitations of this study are: (i) the MLE assumes independent observations whereas daily prices are generally autocorrelated (the lag-1 autocorrelation of these data is 0.898, so the assumption is not met and the goodness-of-fit p-values should be interpreted with caution); (ii) only 86 observations from a single period are used, so the power of the goodness-of-fit tests is limited and the results may not hold for other periods; and (iii) the model describes the price distribution and does not forecast prices.

4. CONCLUSION

The analysis of daily gold prices from 2 February to 1 June 2026 shows a mean of Rp2,634,231.30, a minimum of Rp2,380,281 and a maximum of Rp2,884,960. Parameter estimation by MLE with numerical optimisation using the Quasi-Newton BFGS algorithm gives the Birnbaum-Saunders parameter estimates: shape parameter $\hat{\alpha}$=0.0400 and scale parameter $\hat{\beta} = 2{,}632{,}125.38$. The goodness-of-fit tests with the parametric bootstrap correction (B = 2000) give a Kolmogorov-Smirnov statistic of 0.0775 with a p-value of 0.2424 and an Anderson-Darling statistic of 0.6851 with a p-value of 0.0720. Because both p-values are > 0.05, there is insufficient evidence to reject the fit of the Birnbaum-Saunders distribution to daily gold prices, although the Anderson-Darling p-value, which is only slightly above 0.05, indicates that the fit in the tails should be interpreted with caution. These results are also supported by the visual evaluation in Figure 1, carried out in two ways. First, the density histogram was compared with the theoretical PDF to assess the agreement in location, spread and skewness; this comparison is sensitive to differences in the shape and peak of the distribution. Second, the empirical distribution function (ECDF) was compared with the theoretical CDF; the largest vertical distance between the two curves is the K-S statistic (D_n = 0.0775), so this comparison is sensitive to the largest cumulative deviation, which usually occurs near the centre of the distribution. Departures in the tails (very low or very high prices) are hard to see in either plot and are better detected by the A-D test, which gives more weight to the tails; this is consistent with the A-D p-value being lower than the K-S p-value.

It can therefore be concluded that the Birnbaum-Saunders distribution is suitable for modelling the distribution of daily gold prices from 2 February to 1 June 2026, with the caveat that this conclusion rests on the assumption that daily prices are independent. The comparison with the Normal, Lognormal, Gamma and Weibull distributions shows that the Birnbaum-Saunders distribution has the smallest AIC and BIC, but it is equivalent to the Normal, Lognormal and Gamma distributions (AIC differences below 2) and better than the Weibull distribution. The parameter $\hat{\alpha}$ = 0.0400 indicates relatively small price fluctuations (about 4% of the mean), and the model quantiles give a 90% price range of Rp2,464,542.45 to Rp2,811,103.55, which market participants can use as a simple benchmark.

Future studies are suggested to use longer periods and compare several periods; treat daily prices as dependent data, explore more flexible variants of the Birnbaum-Saunders

distribution (Martínez-Flórez et al., 2019); and link the results to risk measures such as Value at Risk and to forecast evaluation.

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