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Praxeological Analysis of Epistemological Obstacles in Merdeka Curriculum Class XI Advanced Mathematics Textbook

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ABSTRACT

Mastery of complex numbers is paramount in advanced mathematics education. This topic is essential to understanding the Fundamental Theorem of Algebra, which guarantees solutions for high-degree polynomials. Furthermore, the introduction of complex numbers represents a vital structural evolution that unifies algebraic operations with geometric transformations, serving as the definitive foundation for higher-order mathematical analysis. Given this significance, this study analyzes potential epistemological obstacles in the presentation of complex numbers in the Grade XI Advanced Mathematics textbook of the Merdeka Curriculum through a praxeological framework. This qualitative study employed document analysis based on the Anthropological Theory of the Didactic (ATD). The research object was the introductory section on complex numbers in the official textbook published by the Indonesian Ministry of Education, Culture, Research, and Technology in 2021. The analysis systematically focused on the four praxeological components presented in the textbook: task types, techniques, technology, and theory. The findings indicate a significant structural imbalance; the didactic material is overwhelmingly dominated by the praxis block, while the theoretical justifications within the logos block appear severely limited. Consequently, the knowledge structure tends to be rigidly procedure-oriented, and the epistemic model is predominantly externalist, offering students ready-made rules rather than opportunities for meaning-construction. These didactic characteristics may inevitably create deep epistemological obstacles that hinder students' conceptual understanding and spatial-algebraic transition to the complex domain. Therefore, the study recommends that future textbook development adopt a more conceptually coherent, internalist didactic design, fostering exploratory environments that adequately support the theoretical underpinnings of complex numbers.

Keywords: Complex Numbers; Praxeology; Textbook Analysis; Mathematics Book

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INTRODUCTION

Complex numbers constitute an extension of the real number system that possesses its own algebraic structure and encompasses field properties that expand the resolution space of mathematical problems (Nuraida, 2017). Within the context of the Merdeka Curriculum, the official Advanced Mathematics textbook for Phase F presents the topic of complex numbers covering arithmetic operations, conjugates, moduli, arguments, as well as the representation of complex numbers on the complex plane (Masta et al., 2021). The mastery of this concept is highly critical in advanced mathematics education, as it provides the essential framework for the Fundamental Theorem of Algebra, guaranteeing the existence of solutions for higher-degree polynomial equations (Salo, 2024, p. 5). Furthermore, expanding into the complex number system transcends mere algebraic convenience; it introduces a vital structural evolution that unifies algebraic operations with geometric transformations, serving as the definitive foundation for higher-order mathematical analysis (Stewart & Tall, 2018). Consequently, complex numbers have become a crucial subject matter in advanced mathematics education. Nonetheless, complex numbers frequently pose a significant challenge as they demand an understanding of mathematical objects that cannot be fully conceptualized through students' prior experiences with real numbers.

Mutambara and Tsakeni (2022) demonstrated that learners can encounter cognitive obstacles when studying the concept of complex numbers, particularly when the conceptual meaning has not been deeply established. These cognitive obstacles primarily emerge because students instinctively overgeneralize real-number properties and reduce the imaginary unit to a mere symbolic variable, lacking the geometric and structural comprehension required to meaningfully transition into the complex domain. Veith and Bitzenbauer (2021) also highlighted that the complex unit i represents a challenging concept in mathematics education because it is often understood merely as a formal symbol. Furthermore, Suripah and Sthephani (2017) indicated that mathematical understanding and reasoning regarding the complex number system remain a challenge, as students frequently rely on rigid, mechanical procedures and struggle to demonstrate mathematical flexibility when solving complex equations. This overreliance is heavily perpetuated by traditional instructional approaches that prioritize algorithmic rules at the expense of geometric visualizations (Seloane et al., 2025). Recognizing that such procedural-focused teaching is a primary catalyst for students' cognitive obstacles, the instruction of complex numbers must fundamentally shift toward constructing conceptual meaning by integrating algebraic symbols with their geometric visualizations.

In mathematics education, textbooks do not merely serve as a collection of materials and exercises, but they also represent how a mathematical concept is organized and presented to students. Fan et al. (2013) explained that research on mathematics textbooks is essential since textbooks are closely linked to curriculum development and implementation. In line with this, Rezat et al. (2021) asserted that textbooks and mathematics curriculum resources can act as instruments for change in learning. Accordingly, an analysis of textbooks is required to examine how the concept of complex numbers is constructed and potentially understood by students.

Various studies have utilized the praxeological framework to analyze the presentation of mathematical content in textbooks. Utami et al. (2024) analyzed the presentation of the function concept in Indonesian mathematics textbooks and demonstrated the importance of considering the justification of techniques in material presentation. Similar studies were conducted by Rizqi et al. (2021) on set theory, as well as Abung et al. (2025) on the concept of fractions in elementary school mathematics textbooks, showing that the praxeological model can be employed to examine the interconnectedness among tasks,

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techniques, technologies, and theories within textbook presentations. Additionally, Yunianta et al. (2023), Hendriyanto et al. (2023), along with Rahayu et al. (2022) demonstrated that the analysis of task types, techniques, technologies, and theories can be used to uncover the organization of mathematical knowledge in textbooks. However, studies specifically addressing the introduction of complex numbers in the Merdeka Curriculum Advanced Mathematics textbook through a praxeological framework remain limited.

Wijayanti and Winsløw (2017) utilized the praxeological reference model to analyze mathematical practices in textbooks, making this framework highly relevant for examining how mathematical organization is constructed within a learning resource. Bosch and Gascón (2014) also explained that praxeology serves as the fundamental unit of analysis in the Anthropological Theory of the Didactic (ATD) because it connects task types, techniques, technologies, and theories. Therefore, praxeology was selected for this study as it allows for an analysis of the balance between procedural activities and the conceptual foundations in the presentation of complex numbers (Bosch & Gascón, 2014). This choice aligns with the broader epistemological programme in mathematics education, which shifts the research focus away from individual psycho-cognitive learning mechanisms toward the institutional and ecological structures that condition how mathematical organizations are actually transposed and taught in schools (Gascón, 2024).

Quoted from Working with the Anthropological Theory of the Didactic in Mathematics Education: A Comprehensive Casebook by Bosch et al. (2019), a praxeology is a cluster of four interrelated components (T,τ,θ,Θ) which is further divided into two blocks: the praxis block and the logos block. The praxis block consists of the ordered pair of Tasks (T) and their corresponding techniques (τ) to resolve all kinds of tasks. Meanwhile, the logos block consists of the ordered pair of Technology (θ) to justify the techniques and theory (Θ) to explain the technology. Subsequently, the description of each component can be seen in **Table 1**.

Table 1. Components of Praxeological Analysis

Type of Task (T)	**Technique (τ)**	**Technology (θ)**	**Theory (Θ)**
Various types of human activities, which can be small or large, and are distinct from a "job".	Possible methods or ways used to resolve the presented tasks.	The foundation for understanding why a certain technique is used to resolve a type of task.	Something that can control, prove, and generate the technology being used.

Within the praxeological framework, it is vital to get the balance between the praxis and logos elements in order to effectively present mathematical content. The fundamental role of complex numbers in advanced mathematics highlights is of course a case in point and requires a detailed investigation into the epistemological barriers that may emerge during the learning process. This study therefore looks at the way the material is presented in the Grade XI Advanced Mathematics textbook of the Kurikulum Merdeka with a view to assessing the extent to which the praxeological organization and the coherence of the tasks presented will either enable or give rise to epistemological barriers in the students' construction of the concept of complex numbers. Rather than just applying the praxeological framework to a new topic, the study makes an analytical contribution in terms of demonstrating how the pedagogical design of a textbook can be used to anticipate or indeed inadvertently create epistemological barriers in the students' development of the concept.

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METHOD

Research Design and Setting

This study is a qualitative research employing a document analysis approach. The analysis was conducted using the framework of the Anthropological Theory of the Didactic (ATD), specifically the concept of praxeology which encompasses task types (T), techniques (τ), technologies (θ), and theories (Θ). The object of this research is the presentation of complex number material in Chapter 1 Section A of the Advanced Mathematics textbook for Senior High School Grade XI, published by the Center for Textbooks, Board of Standards, Curriculum, and Educational Assessment, Ministry of Education, Culture, Research, and Technology in 2021. The selection of this section was purposive, as it captures the initial epistemological transition into complex domain by addressing the conceptual limitations of the real number set. The analytical procedures, encompassing document analysis and data evaluation, were executed between February to May 2026.

Participants, Instruments, and Procedures

The analysis was focused specifically on Chapter 1, Section A (Introduction to Complex Numbers), covering pages 4 to 14 of the textbook. Within this range, a total of 19 units of analysis containing activities that are directly concerned with the initial introduction of the concept of complex numbers were identified, comprising 2 exploration tasks, 4 formal definitions, 6 worked examples, 6 '*Mari Mencoba'* (try-it-out) exercises, and 1 communication task. The units of analysis do not include the concept maps, the chapter introductions and the activities which are not directly involved in the initial construction of the concept of complex numbers (in order to keep the analysis focused). The units of analysis also do not include the general exercises as these are assumed to be primarily concerned with skill practice; that said, this section was still carefully scrutinized to ensure that there were no justifications or contextual variations that would in any way change the interpretation of the results. The analytical procedure proceeded in a number of stages: reading through the whole presentation of the material, identifying the types of tasks, identifying the techniques for completing the tasks, looking at the justifications (the technology), identifying the underlying theory (the logos) and then finally interpreting the potential for epistemological obstacles in terms of the balance between the praxis and logos blocks.

The task types were classified into six categories (T_1–T_6) based on their distinct mathematical objectives and the order in which the corresponding concepts and activities are presented in the textbook. Thus, the classification follows the sequence of the textbook's content rather than imposing an externally determined conceptual progression. T_1 covers tasks involving the identification of non-real number forms through square roots of negative numbers. T_2 includes tasks requiring students to identify and write the real and imaginary components of a complex number in its standard form, $a + ib$. T_3 comprises tasks involving the solution of quadratic equations with non-real solutions. T_4 concerns the geometric representation of complex numbers by locating their real and imaginary components on the complex plane. T_5 encompasses tasks requiring the conversion of complex numbers from Cartesian form into polar and exponential forms. Finally, T_6 includes tasks involving the verification of equality between two complex numbers by comparing their corresponding real and imaginary components. This classification preserves the sequence of the mathematical content as presented in the textbook, from the first identified task type (T_1) to the last (T_6).

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To distinguish between the logos block components, Technology (θ) was operationalized as the immediate mathematical discourse or explicit rules directly justifying the technique (e.g., stating $i^2 = -1$ to solve an equation), whereas Theory (Θ) was defined as the broader, often implicit, structural foundation that validates the technology (e.g., the extension of the algebraic field properties). The criteria for potential epistemological obstacles were strictly operationalized and distinguished from didactical obstacles, as the mere fact that a textbook explanation has limitations does not in itself imply that students are in fact encountering epistemological obstacles. The analysis is therefore presented in a clear line of evidence: starting with the identification of the relevant quotes/tasks in the book, looking at the limitations of the knowledge constructed in that context, and then moving on to the evaluation of new contexts in which that knowledge has the potential to break down.

Data Analysis

Data validity was supported through an expert review conducted by an expert in the didactics of mathematics education. The review covered three aspects: (1) the scope and selection of the units of analysis, particularly their relevance to the initial introduction of complex numbers; (2) the classification of task types and praxeological components (T, τ, θ, and Θ), including their consistency with the textbook content and the analytical framework; and (3) the overall both preliminary and final manuscript draft. The expert's feedback was considered in reviewing the analysis and validate the manuscript to improve the accuracy and consistency of the findings

RESULTS

The results of this study consist of an analysis of potential epistemological obstacles in the Grade XI advanced mathematics textbook on complex numbers, conducted based on praxeology. The analysis encompasses a brief overview of the praxeological components in the textbook, the characteristics of the knowledge system within the textbook, and the epistemological characteristics in the introduction of complex numbers. In the analysis section of the praxeological components, the didactic design for introducing the concept of complex numbers was identified as consisting of six task types, subsequently denoted as $T_1 - T_6$. The description of each task can be seen in **Table 2**.

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Table 2. Praxeological Components Based on $T_1 - T_6$ in Complex Number Material

CODE	TYPE OF TASK (T)	TECHNIQUE (τ)	TECHNOLOGY (θ)	THEORY (Θ)
T_1	Identifying non-real number forms through the square root of negative numbers.	Classifying numbers by checking for the presence of square roots of negative numbers.	In real numbers, the square root operation is only defined for non-negative numbers.	Extension of the algebraic structure from real to complex to maintain the closure of algebraic operations. Complex numbers emerge from the inability of the real system to solve all polynomial problems.
T_2	Writing and identifying $Re(z)$ and $Im(z)$ components.	Identifying the values of a and b from the standard form $a + ib$.	The form $a + ib$ serves as an operational representation that allows complex numbers to be viewed through the separation of real and imaginary components.	Complex numbers are viewed as ordered pairs (a, b) with specific operational rules, such that the real and imaginary parts comprise two components of a single unified algebraic number.
T_3	Solving quadratic equations that have non-real solutions.	Substituting the value of $\sqrt{-1}$ with i to obtain the solution.	The imaginary unit i acts as an algebraic element that satisfies the fundamental relation $i^2 = -1$	The field extension from real to complex preserves the properties of algebraic operations.
T_4	Drawing the position of numbers on the complex plane.	Placing the sets of $Re(z)$ and $Im(z)$ on the axes to construct a plane.	Every complex number has a one-to-one correspondence with an ordered pair on the Cartesian plane.	The complex set is modeled as a two-dimensional vector space over the reals, so that complex operations have a geometric interpretation.
T_5	Converting Cartesian form into polar and exponential forms.	Using the modulus$r = \sqrt{x^2 + y^2}$ and the relationship between trigonometry and variables. Using Euler's formula on the polar form of complex numbers.	The position of a complex number can be represented through the modulus and the angle. The exponential function shares a structural relationship with trigonometric functions through Euler's formula.	Analysis of complex variable functions regarding elementary transcendent functions and vector representation in polar coordinates.
T_6	Verifying the equality of two complex numbers.	Comparing the equality of the real and imaginary parts separately.	Component-wise equality property, wherein two complex objects are identical only if all of their corresponding parts are equal.	Definition of equality within the algebraic structure of the complex number field.

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The findings from the Grade XI Merdeka Curriculum mathematics textbook on complex numbers show that the design of the teaching material is very much in favor of the praxis block, with the logos block being restricted to the provision of procedural justifications or to the researcher's own reconstruction of the theoretical underpinnings rather than the presentation of formal proofs appropriate for the secondary school level. **Table 2** sets out the reconstructed technology and theory components to illustrate the full praxeological structure of the material but the textbook itself presents the material in the form of ready-made rules and definitions. In T_2 for instance the textbook explicitly gives Definition 1.1 (p. 6) for the real and imaginary parts but makes no explicit reference to the underlying algebraic field structure of the complex numbers. In T_3 (pp. 7-8) the solution technique is to substitute without any explicit justification as to why the real algebraic operations can be extended to the complex numbers. In T_4 the Argand plane is introduced via an explicit 1-to-1 mapping to the Cartesian plane (pp. 8-10), the theoretical link between the vector space properties and the geometric addition of complex numbers being entirely implicit. In T_5 Euler's formula (Rumus Euler) (p. 12) is simply stated as a given identity (Definisi 1.4) with no intuitive or appropriate heuristic justification for the connection between the exponential and trigonometric forms. In T_6 (pp. 13-14) component-wise equality is presented as a procedural rule for verification rather than as a consequence of the equality of algebraic structures. The procedural nature of T_2 is illustrated in **Figure 1**.

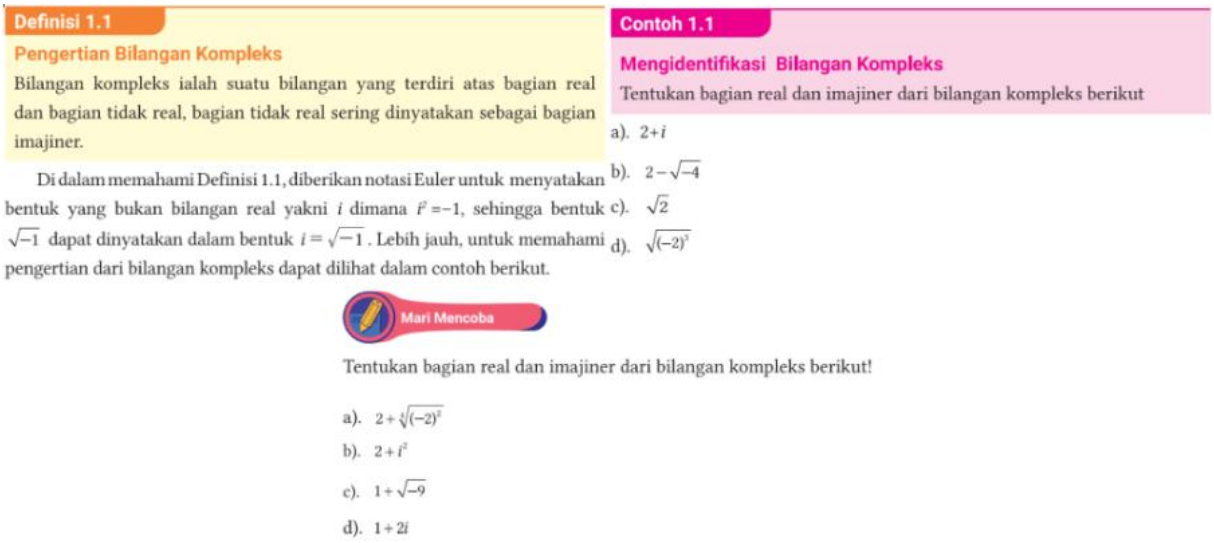

Figure 1. Presentation of T_2 on complex numbers in the textbook (Masta et al., 2021, p. 6).

Based on the details of the analysis of the praxeological characteristics of T above, the knowledge system constructed can subsequently be identified by referring to the question: "Does the praxeology taught in Section A form a conceptually coherent knowledge system, or does it focus solely on the procedural?". As a reference, the differences between conceptual-oriented and procedural-oriented approaches regarding the primary focus, the relationship between T, how knowledge is introduced, the role of students, and validation can be observed in **Table 3**.

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Table 3. Differences between Conceptual-Oriented and Procedural-Oriented Approaches

ASPECT	CONCEPTUAL-ORIENTED	PROCEDURAL-ORIENTED
Primary Focus	Interconnectedness of mathematical ideas and their meanings	Sequential order of procedures and techniques
Relationship between T	Connected based on mathematical ideas	Connected based on the ease of procedures
Knowledge	Through conceptualization	Through memorization
Role of Students	Constructing meaning	Repeating rules and patterns
Validation	Rational-conceptual	Empirical-perception and reproduction

In the **Table 3**, the conceptual-oriented approach views ideal learning as focusing on the interconnectedness of mutually reinforcing ideas so that students understand the reasoning behind each procedure. Conversely, the procedural-oriented approach emphasizes sequencing based on content from simple to complex levels. Referring to **Table 3**, the structure of knowledge constructed in the textbook appears systematic, where knowledge begins with the introduction of non-real forms (T_1), dissecting the components of complex numbers (T_2), utilizing imaginary unit manipulation to solve quadratic equations (T_3), drawing complex numbers on the Argand plane (T_4), transforming the forms of complex numbers (T_5), and concluding with the equality of complex numbers (T_6). Furthermore, **Table 4** demonstrates the characteristics of the relationships between the types of tasks and the concept of number structure extension, the transitional nature between the types of tasks, and their coherence characteristics.

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Table 4. Characteristics of Relationships and Transitions Between Task Types

PROGRESSION	RELATIONSHIP WITH NUMBER STRUCTURE EXTENSION	NATURE OF TRANSITION BETWEEN TASKS	COHERENCE
$T_1 - T_2$	There is a transition of meaning from the impossibility of real number solutions to the need for extending the number structure (the presence of the symbol i).	The transition is facilitated through exploration activities, making it conceptual in nature.	Conceptual-oriented
$T_2 - T_3$	In this transition, the resulting knowledge regarding the extension of the number structure is used to solve equations.	Procedural, where students are directly guided to substitute the definition of $i = \sqrt{-1}$ to solve the problem.	Tends to be procedural-oriented
$T_2 - T_4$	Provides a geometric visualization of the newly introduced abstract number.	Views complex numbers as ordered pairs and analogizes them to Cartesian coordinates.	Conceptual-oriented
$T_3 - T_4$	There is a conceptual leap from finding solutions to quadratic equations to geometric representation.	The transition follows a hierarchy of complexity, where T_4 is introduced without involving the development of ideas from T_3.	Procedural-oriented
$T_4 - T_5$	Shifts the representation from the position of real and imaginary axes to angles and distances.	The connection to trigonometry is continued by developing ideas from the previous task, but the connection to exponents is presented as a ready-made formula.	A mixture of conceptual and procedural
$T_5 - T_6$	The focus shifts from representation to technical rules for verifying number equality.	The transition follows a hierarchy of complexity, and the rule is provided as a ready-made method.	Procedural-oriented
$T_1 - T_6$	Overall, there is a linear narrative spanning from the initial problem to technical operations.	Transitions between tasks tend to follow the ease of learning sequence rather than the progressive development of mathematical ideas.	Coherence prominently features a procedural-oriented approach

Broadly speaking, the sequence of the given types of tasks successfully develops knowledge from basic concepts to the student's operational skills. Upon closer inspection, the introduction of the material content successfully begins with the development of mathematical ideas from the limitations of real numbers in solving algebraic problems toward the necessity of extending the number structure. However, when examined more deeply, $T_1 - T_6$ exhibit a hierarchy of skills rather than a hierarchy of ideas; that is, the instructional sequence of the types of tasks is determined based on the ease of the techniques used to resolve the tasks. Several transitions from one task to the next do not reflect the progressive development of mathematical ideas that connect the concepts of complex numbers. Consequently, knowledge is acquired by students through a process of observation and imitation rather than as a result of logical generalization. This leads students to validate the correctness of the techniques used solely through examples and visual observations rather than based on logical argumentation. This state of affairs ensures that knowledge is constructed based on practice rather than reflection. Based on this analysis, the constructed knowledge system forms a procedural-oriented approach.

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In addition to addressing the characteristics of knowledge, this analysis investigates the epistemological features of the introduction to complex numbers using the Anthropological Theory of the Didactic (ATD) framework. In this didactic context, the externalist–internalist framing specifically concerns the epistemic opportunities for meaning-construction provided by the textbook's instructional design, rather than making empirical claims about student cognitive behavior or learning outcomes. An externalist presentation introduces definitions, rules, and representations as established facts, while an internalist presentation enables students to construct mathematical meaning and deduce underlying properties through active engagement. Each task type (T_1–T_6) is examined in terms of its presentation format and task demands to evaluate whether the textbook supports reflective-conceptual construction or primarily delivers content externally. **Table 5** summarizes this epistemic classification across all task types.

Table 5. Analysis of Epistemological Characteristics

PRAXEOLOGY	TYPE OF ACTIVITY	FORM OF PRESENTATION	EPISTEMIC MODEL
T_1: Non-Real Forms	Differentiating between real and non-real number forms.	Knowledge is introduced through a direct rule stating that in real numbers $\sqrt{x}$ is only valid if x is non-negative, and students are then prompted to explore independently.	Transition from externalist to internalist
T_2: Definition of Complex Numbers	Students determine the real and imaginary parts of numbers.	Knowledge is introduced through a definition provided directly by the author.	Externalist (The author directly provides the definition)
T_3: Solving Quadratic Equations	Students find solutions to quadratic equations.	Knowledge is constructed through the observation of examples.	Externalist
T_4: Geometric Visualization	Drawing complex numbers by viewing them as ordered pairs (a, b)	Knowledge is directly delivered by stating complex numbers as ordered pairs and providing their visual representations.	Externalist
T_5: Geometric Visualization	Converting Cartesian form into polar and exponential forms.	The connection to the polar form is derived based on visual representation, but the exponential form is given directly.	Externalist, though it possesses potential for conceptual reflection
T_6: Verifying the Equality of Two Complex Numbers	Verifying the equality of two complex numbers.	Rules are written directly without further explanation or reflection.	Externalist

Based on the description in **Table 5**, students' thinking processes tend to be dominated by an externalist epistemic model, meaning that opportunities for students to engage in exploration and actively construct knowledge remain limited. Students are not positioned as co-constructors who possess the autonomy to carry out mathematical processes and meaning-making. This can trigger the emergence of potential epistemological learning obstacles, such as in the presentation of T_2, which may cause students to merely memorize definitions without grasping the underlying meaning; T_3, which may lead students to view i merely as a substitute symbol for $\sqrt{-1}$; and T_4, which may result in students failing to comprehend the urgency of studying the geometric representation of imaginary numbers. Although the transition to polar form in T_5 possesses the potential for students to reflectively construct knowledge through geometric relationships, this opportunity is abruptly halted when the exponential form is presented as a given identity, as shown in **Figure 2**.

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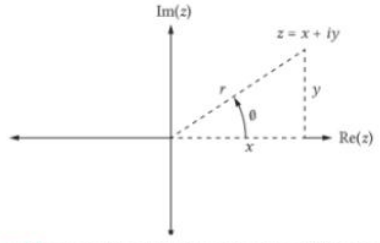

Gambar 1.3 Representasi Bentuk Polar Bilangan Kompleks pada Bidang Kompleks

Perhatikan Gambar 1.3, dapat dilihat terdapat sudut antara sumbu Re(z) dan garis penghubung titik z dan titik 0. Bentuk ini memberikan ide penggunaan aturan trigonometri untuk merepresentasikan bilangan kompleks. Berdasarkan sifat Trigonometri diperoleh $\cos\theta = \frac{x}{r}$ dan $\sin\theta = \frac{y}{r}$, akibatnya $x = r\cos\theta$ dan $y = r\sin\theta$ dengan $r = \sqrt{x^2 + y^2}$. Akibatnya, diperoleh $z = x + iy = r\cos\theta + ir\sin\theta = r(\cos\theta + i\sin\theta)$. Bilangan kompleks $z = r(\cos\theta + i\sin\theta)$ disebut sebagai bentuk polar dan dapat dinyatakan juga sebagai $z = (r, \theta)$ atau $z = rcis\theta$.

Definisi 1.3

Bentuk Polar Bilangan Kompleks

Misalkan x, y bilangan real maka bilangan kompleks $z = x + iy$ dapat dinyatakan dalam bentuk polar yakni,

$z = r(\cos\theta + i\sin\theta)$ dengan $r = \sqrt{x^2 + y^2}$, $x = r\cos\theta$ dan $y = r\sin\theta$.

Untuk memahami bentuk polar dari bilangan kompleks dapat dilihat dalam contoh berikut.

Selain itu, terdapat suatu identitas yang menyatakan $e^{i\theta} = \cos\theta + i\sin\theta$. Dengan memanfaatkan identitas tersebut, maka bilangan kompleks $z = r(\cos\theta + i\sin\theta)$ bisa dinyatakan dalam bentuk $z = r(\cos\theta + i\sin\theta) = re^{i\theta}$. Bentuk bilangan kompleks $z = re^{i\theta}$ disebut sebagai bentuk eksponen. Adapun definisi bentuk eksponen bilangan kompleks dinyatakan sebagai berikut.

Definisi 1.4

Bentuk Eksponen Bilangan Kompleks

Misalkan $z = r(\cos\theta + i\sin\theta)$ adalah bentuk polar bilangan kompleks maka z dapat dinyatakan dalam bentuk eksponen yakni,

$$z = re^{i\theta}$$

dengan $e^{i\theta} = \cos\theta + i\sin\theta$.

Figure 2. Presentation of polar and exponential forms of complex numbers in the textbook (Masta et al., 2021, pp. 10–12).

DISCUSSION

Based on the praxeological structure and the subsequent detailed analysis of tasks T_1 through T_6, the textbook's instructional design exhibits a clear procedural dominance. This reinforces the findings of Utami et al. (2024), which state that the justification of techniques in Indonesian mathematics textbooks is often less profound. In line with this, the dominance of the praxis block (tasks and techniques) over the logos block (technology and theory) in this study supports the findings of several praxeological studies on mathematics textbooks, indicating that technical justification and theoretical support in textbook presentations still require deeper consideration (Utami et al., 2024; Chandra et al., 2025; Fitriasari et al., 2025). Furthermore, the lack of instilled conceptual meaning in this material potentially gives rise to didactic obstacles similar to those identified in the research by Ulfa, et. al (2021). In their research findings, a teacher's overemphasis on procedural aspects without meaningful construction causes students to be inconsistent in solving problems and easily prone to misconceptions due to merely memorizing routine formulas. To resolve this structural imbalance, learning models must shift toward exploratory environments, such as utilizing GeoGebra Classroom to scaffold representational transitions and effectively mitigate these emerging epistemological obstacles (Raisatunnisa et al., 2026).

As revealed by the analysis of tasks $T_1 - T_6$ in **Table 4**, the textbook predominantly presents mathematical knowledge through an externalist epistemic approach. Except for the exploratory element in T_1 and the conceptual potential identified in T5, most tasks introduce mathematical knowledge through directly provided definitions, rules, or representations. This result strongly echoes recent textbook analyses within the Indonesian curriculum ecosystem. For instance, Panjaitan and Dahlan (2025) uncovered identical issues in Merdeka Curriculum textbooks, where tasks predominantly relied on surface-level perceptual techniques rather than theoretical justification. This structural reliance on superficial verification and visual evaluation also mirrors the cross-country textbook analysis by Nurhikmayati et al. (2025). Their comparative study revealed that Indonesian secondary mathematics textbooks frequently depend on perceptual techniques that restrict students to reading ready-made

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conclusions, consequently generating significantly more epistemological obstacles compared to exploratory frameworks that utilize physical or operational actions. Furthermore, this knowledge system gives rise to various potential learning obstacles, such as students perceiving the geometric representation as a separate entity from the $a + ib$ form, and viewing various topics within complex numbers as distinct and unrelated subjects. Moreover, the lack of an internalist didactic design found as presented in **Table 5** within the Merdeka Curriculum textbooks is further exposed by Fardian et al. (2025). Through a comprehensive praxeological investigation, they demonstrated that modern Indonesian mathematics materials are often non-systemic and lack epistemic coherence, creating structural gaps that fail to facilitate a smooth cognitive transition for students and leading directly to severe didactical and epistemological learning obstacles. When a knowledge system operates with a heavily externalist and procedural orientation, student thinking becomes rigid and susceptible to misconstructions, parallel to the instructional deficiencies highlighted by Ulfa et al. (2021) and the structural learning difficulties mapped out by Mutambara and Tsakeni (2022).

To address these potential obstacles, the textbook should present complex numbers as a coherent mathematical knowledge system in which algebraic and geometric representations are meaningfully interconnected. Rather than treating definitions, properties, and procedures as isolated pieces of information, their presentation should clarify the mathematical reasoning underlying each concept and the relationships among them (Juhaevah et al., 2025; Hendriyanto et al., 2024). In this regard, explicit mathematical explanations and formal representations should be complemented by opportunities for students to explore mathematical relationships, develop justifications, and validate their reasoning. For instance, the introduction of geometric representations should not merely establish the correspondence between the algebraic form $(a + ib)$ and points on the Argand plane, but also explain the mathematical basis of this correspondence and its implications for understanding complex number operations and properties. Such an approach would allow students to engage not only with established mathematical knowledge but also with the reasoning processes through which its meaning and coherence can be understood. Ideally, mathematical knowledge should constitute propositional knowledge in the form of justified true belief, in which mathematical claims are supported by sound reasoning rather than merely presented as established facts (Hendriyanto, 2024). Furthermore, such knowledge should be organized as a coherent system in which mathematical tasks and their underlying justifications are structurally and functionally interconnected, allowing each concept to contribute meaningfully to the development of the broader mathematical organization (Fardian et al., 2025).

CONCLUSION

Based on the praxeological analysis of the Grade XI Merdeka Curriculum Advanced Mathematics textbook, the initial treatment of complex numbers (tasks T_1 to T_6) is overwhelmingly procedural with the praxis block far outweighing the logos block. The progression of tasks is essentially one of routine techniques and the use of ready-made representations with very little emphasis on the underlying theoretical justification or on the connections between the various algebraic, polar and geometric forms. In keeping with the nature of a document-based study, these findings point to potential epistemological learning obstacles that are untested rather than verified as actual causes of student difficulty in the classroom. To address these structural deficiencies the curriculum would need to incorporate concrete task sequences that explicitly link the algebraic forms to the geometrical representations via reflective, internalist tasks which are rooted in theoretical justification. The use of digital tools (e.g., GeoGebra) as an intervention to address these potential obstacles is also untested in this study and would be a good candidate for further empirical investigation.

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The limitation of this study lies in its exclusive focus on the textbook. Based on the framework by Bosch et al. (2019), there is a discrepancy between the intended curriculum (in the textbook) and the attained curriculum (received by the students). Therefore, the potential epistemological obstacles discovered remain predictive and require verification through educational interactions in the classroom. In addition, this study has not dissected the praxeology within the section on complex number operations. Thus, further analysis is still necessary, given that complex number operations entail distinct procedural complexities.

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Data Availability: The materials analyzed in this study are publicly available in the Mathematics Textbook for the Merdeka Curriculum. The source material can be accessed by the public.

Conflict of Interest: The authors declare that they have no conflict of interest related to this study.

Use of Generative AI: Generative AI tools were used to assist with brainstorming and developing research concepts, recommending English word choices and expressions for academic writing, and checking the manuscript's formatting against the publisher's guidelines. The authors reviewed and verified all AI-assisted suggestions and take full responsibility for the final content of the manuscript.

REFERENCES

Abung, M., Putra, A. D., Nurjanah, S., & Astiti, N. Y. (2025). Analisis sajian materi konsep pecahan pada buku teks matematika kelas IV SD berdasarkan prakseologi. *Jurnal Cendekia: Jurnal Pendidikan Matematika*, 9(2), 603–617. https://doi.org/10.31004/cendekia.v9i2.3995

Bosch, M., Chevallard, Y., García, F. J., & Monaghan, J. (Eds.). (2019*). Working with the anthropological theory of the didactic in mathematics education*. Routledge. https://doi.org/10.4324/9780429198168

Bosch, M., & Gascón, J. (2014). Introduction to the anthropological theory of the didactic (ATD). In *Networking of theories as a research practice in mathematics education* (pp. 67–83). Springer. https://doi.org/10.1007/978-3-319-05389-9_5

Chandra, F. E., Suryadi, D., Afgani Dahlan, J., Hayuningrat, S., & Rahman, S. (2025). Derivative in Indonesian textbook curricula: A praxeological analysis of learning obstacles in Indonesian mathematics textbooks. *Eurasia Journal of Mathematics, Science and Technology Education*, 21(5), Article em2627. https://doi.org/10.29333/ejmste/16256

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Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: Development status and directions. *ZDM – Mathematics Education*, 45(5), 633–646. https://doi.org/10.1007/s11858-013-0539-x

Fardian, D., Suryadi, D., & Prabawanto, S. (2025). A praxeological analysis of linear equations in Indonesian mathematics textbooks: Focusing on systemic and epistemic aspect. *Journal on Mathematics Education*, 16(1), 225–254. https://doi.org/10.22342/jme.v16i1.pp225-254

Fitriasari, P., Suhendra, Suryadi, D., & Nurlaelah, E. (2025). Textbook analysis of integers content using the praxeology framework. *Jurnal Elemen*, 11(3), 577–596. https://doi.org/10.29408/jel.v11i3.29859

Gascón, J. (2024). Contributions of the anthropological theory of the didactic to the epistemological programme of research in mathematics education. *ZDM – Mathematics Education*, 56(2), 231–243. https://doi.org/10.1007/s11858-024-01563-1

Hendriyanto, A., Suryadi, D., Dahlan, J. A., & Juandi, D. (2023). Praxeology review: Comparing Singaporean and Indonesian textbooks in introducing the concept of sets. *Eurasia Journal of Mathematics, Science and Technology Education*, 19(2), Article em2229. https://doi.org/10.29333/ehendriyantojmste/12953

Hendriyanto, A. (2024). Desain didaktis materi himpunan: Studi didactical design research dalam pendampingan transposisi didaktis internal bagi calon pendidik profesional [*Doctoral dissertation, Universitas Pendidikan Indonesia*]. UPI Repository. https://repository.upi.edu/119161/

Juhaevah, F., Upu, H., Bernard, & Talib, A. (2025). Trends and recommendation practices in didactical design for mathematics learning: A systematic literature review. *Journal of Scientific Research, Education, and Technology (JSRET)*, 4(4), 2505–2532. https://doi.org/10.58526/jsret.v4i4.954

Masta, A. A., Kristanto, Y. D., Yulfiana, E., & Taqiyuddin, M. (2021). *Matematika tingkat lanjut untuk SMA kelas XI*. Pusat Kurikulum dan Perbukuan.

Mutambara, L. H. N., & Tsakeni, M. (2022). Cognitive obstacles in the learning of complex number concepts: A case study of in-service undergraduate physics student-teachers in Zimbabwe. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(10), Article em2158. https://doi.org/10.29333/ejmste/12418

Nuraida, I. (2017). Sifat lapangan pada bilangan kompleks. *Jurnal Analisa*, 3(1), 70–75. https://doi.org/10.15575/ja.v3i1.1498

Nurhikmayati, I., Darhim, Dahlan, J. A., & Wijaya, T. T. (2025). A comparison of Indonesia and Singapore secondary school textbooks of mathematics: A praxeological analysis of cube and cuboid task design. *European Journal of Mathematics and Science Education*, 6(1), 65–78. https://doi.org/10.12973/ejmse.6.1.65

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Panjaitan, M. A., & Dahlan, J. A. (2025). Materi peluang dalam buku teks matematika Kurikulum Merdeka: Analisis prakseologi. *Teorema: Teori dan Riset Matematika*, 10(2), 267–276. https://doi.org/10.25157/teorema.v10i2.18646

Rahayu, T. G., Herman, T., & Prawiyogi, A. G. (2022). Teori dan teknologi materi pecahan pada buku teks matematika sekolah dasar. *Mimbar Ilmu*, 27(2), 321–332. https://doi.org/10.23887/mi.v27i2.45158

Raisatunnisa, Suryadi, D., Fatimah, S., Priatna, N., Chen, J., & Wicaksono, A. (2026). Geogebra Classroom for overcoming epistemological obstacles and enhancing mathematical literacy: A Bruner perspective theory. *Koordinat Jurnal Pembelajaran Matematika dan Sains*, 7(1), 1–18. https://doi.org/10.24239/koordinat.v7i1.205

Rezat, S., Fan, L., & Pepin, B. (2021). Mathematics textbooks and curriculum resources as instruments for change. *ZDM – Mathematics Education*, 53(6), 1189–1206. https://doi.org/10.1007/s11858-021-01309-3

Rizqi, M. M., Wijayanti, D., & Basir, M. A. (2021). Analisis buku teks matematika materi himpunan menggunakan model prakseologi. *Delta: Jurnal Ilmiah Pendidikan Matematika*, 9(1), 57–66. https://doi.org/10.31941/delta.v9i1.1226

Salo, M. (2024). *Complex analysis 1: Lecture notes, spring 2024*. Department of Mathematics and Statistics, University of Jyväskylä.

Seloane, M., Ramaila, S., & Ndlovu, M. (2025). Harnessing GeoGebra as a modelling tool to mitigate undergraduate engineering mathematics students' misconceptions and errors associated with complex numbers. *Pythagoras*, 46(1), Article a825. https://doi.org/10.4102/pythagoras.v46i1.825

Stewart, I., & Tall, D. (2018). *Complex analysis: The hitch-hiker's guide to the plane*. Cambridge University Press.

Suripah, S., & Sthephani, A. (2017). Kemampuan berpikir kreatif matematis mahasiswa dalam menyelesaikan akar pangkat persamaan kompleks berdasarkan tingkat kemampuan akademik. *PYTHAGORAS: Jurnal Pendidikan Matematika*, 12(2), 149–160. https://doi.org/10.21831/pg.v12i2.16509

Ulfa, N., Jupri, A., & Turmudi. (2021). Analisis hambatan belajar pada materi pecahan. *Research and Development Journal of Education*, 7(2), 226–236. http://dx.doi.org/10.30998/rdje.v7i2.8509

Utami, N. S., Prabawanto, S., & Suryadi, D. (2024). How do Indonesian students learn function concepts? A praxeological analysis of textbook. *Journal on Mathematics Education*, 15(2), 451–472. https://doi.org/10.22342/jme.v15i2.pp451-472

Veith, J. M., & Bitzenbauer, P. (2021). Two challenging concepts in mathematics education: Subject-specific thoughts on the complex unit and angles. *European Journal of Science and Mathematics Education*, 9(4), 244–251. https://doi.org/10.30935/scimath/11251

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Wijayanti, D., & Winsløw, C. (2017). Mathematical practice in textbooks analysis: Praxeological reference models, the case of proportion. *REDIMAT – Journal of Research in Mathematics Education*, 6(3), 307–330. https://doi.org/10.17583/redimat.2017.2078

Yunianta, T. N. H., Suryadi, D., Dasari, D., & Herman, T. (2023). Textbook praxeological-didactical analysis: Lessons learned from the Indonesian mathematics textbook. *Journal on Mathematics Education*, 14(3), 503–524. https://doi.org/10.22342/jme.v14i3.pp503-524

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