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Pedagogical Modalities of Pre-Service Mathematics Teachers in Designing Hypothetical Learning Trajectories

Nidya Ferry Wulandari[1] · Iqbal Ramadani[1*]

[1]Universitas Islam Negeri Sunan Kalijaga, Yogyakarta, Indonesia
* Corresponding author: iqbal.ramadani@uin-suka.ac.id

ABSTRACT

Designing a hypothetical learning trajectory (HLT) requires pre-service mathematics teachers to connect learning objectives and tasks with how students might think as they work. This study examined the pedagogical modalities reflected in their lesson plans, focusing on how they organized mathematical content and activities, anticipated student thinking, and planned instructional responses. Using a qualitative instrumental case study design, two researchers independently assessed 41 mathematics lesson plans and compared their ratings and interpretations. A whole-class discussion provided context for the difficulties the pre-service teachers reported. The planned activities followed a logical sequence in 38 lesson plans, while the activities in 21 plans were judged to support the stated learning objectives. Twenty-two plans documented a specific student strategy, response, error, or difficulty that might arise during a task. Five plans specified a concrete instructional response. During the class discussion, some pre-service teachers reported difficulty imagining how students would respond and explained that they expected to address difficulties as they arose in class. Their accounts offer one possible explanation for why specific follow-up actions were rarely documented. The findings suggest that lesson-planning assignments could ask pre-service teachers to connect a mathematical task to a plausible student response and specify a follow-up that advances the learning objective. The planned response can then be adapted to the reasoning students actually show.

Keywords: hypothetical learning trajectory; instructional planning; pedagogical modalities; pre-service mathematics teachers; student thinking

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INTRODUCTION

The curriculum serves as a guide for teachers in planning, implementing, and evaluating instruction. During the transition from the official curriculum to the intended curriculum, teachers must understand the content and processes underlying the official curriculum and adapt them to students' levels and needs, as well as to available resources, such as textbooks, curriculum guides, and lesson plans. Teachers then design learning activities based on the intended curriculum (Tran & O'Connor, 2024). For pre-service mathematics teachers, this involves translating a curricular learning outcome into a lesson objective, selecting a mathematical task that serves that objective, and anticipating how students might approach the task.

Teachers require both formal and practical competencies to conceptualize instruction. Curricular knowledge includes an understanding of curriculum materials and the relationships between topics within and across subjects. Curricular reasoning refers to the process of connecting curriculum content to decisions about planning, implementing, and reflecting on instruction. Curricular noticing concerns what teachers attend to in curriculum materials, how they interpret it, and how they respond when planning instruction (Tran & O'Connor, 2024). In mathematics lesson planning, these processes inform decisions about which tasks to select and how to prepare for their use in the classroom (Hammer & Ufer, 2023).

A framework for designing instruction and facilitating student learning is the hypothetical learning trajectory (HLT). Teachers can use an HLT to design instructional activities suited to students' prior knowledge and anticipated ways of thinking (Lantakay et al., 2023). Simon (1995) identifies three components of an HLT. These are a learning goal, the activities intended to support that goal, and a hypothesis about how students' thinking may develop as they engage with those activities. The present study also examines planned follow-up responses. For example, a lesson plan may anticipate that students will compare fractions by looking only at their denominators. A follow-up specifies what the teacher could ask or show in response to that reasoning. One challenge in lesson planning is aligning learning indicators with the intended competencies. Agustina and Jumadi (2020) documented this problem in lesson plans prepared by Indonesian physics teacher education students. For mathematics lesson planning, alignment gives teachers a basis for deciding what a task should help students learn and what to look for in their responses.

In this study, pedagogical modalities refer to the pedagogical and professional resources that pre-service mathematics teachers draw on when designing instruction. The study examines how these resources are reflected in their teaching modules. The pedagogical dimension concerns the choice of instructional models and how their steps are represented in the planned activities. The professional dimension concerns the alignment of mathematical content with curricular learning outcomes, as well as the sequence and completeness of that content. The HLT framework guides the analysis of learning objectives, activities, and anticipated student thinking.

Courses on curriculum and instructional planning provide opportunities for pre-service mathematics teachers to develop these aspects of lesson design. These courses aim to develop an understanding of mathematics curricula at various educational levels and guide pre-service teachers in planning instruction that aligns with these curricula. The present study examines teaching modules developed in the Curriculum Analysis and Mathematics Instruction Planning course at UIN Sunan Kalijaga

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Yogyakarta. These modules make it possible to examine the links between curricular learning outcomes, lesson objectives, planned tasks, and anticipated student responses within a single lesson design.

In Indonesia, Fiangga et al. (2021) analyzed HLTs designed by pre-service mathematics teachers in a Realistic Mathematics Learning course and reported difficulties in anticipating misconceptions and planning responses to students' thinking. Rianasari and Guzon (2024) designed a learning trajectory to support pre-service mathematics teachers in creating and implementing realistic mathematics tasks. Building on these studies, the present study examines HLT design in a curriculum and instructional planning course. It considers how the components of an HLT are documented alongside the choice of instructional models and the alignment and sequencing of mathematical content in teaching modules.

This study investigates the pedagogical modalities reflected in the teaching modules developed by pre-service mathematics teachers. It addresses two research questions.

1. Which instructional models are selected, and how are mathematical content, learning objectives, and activities organized in the teaching modules?
2. How do the teaching modules document anticipated student thinking and planned instructional responses?

The contribution of this research lies in profiling how pre-service mathematics teachers design HLTs. By examining anticipated student thinking alongside planned follow-up, the study can help teacher educators identify where pre-service teachers need support in turning a prediction about students' reasoning into an instructional decision.

METHOD

This study used a qualitative instrumental case study design (Stake, 1995) to examine how pre-service mathematics teachers documented hypothetical learning trajectories (HLTs) in their lesson plans. The case was bounded by one cohort in a mathematics education program at an Indonesian university. The documents analyzed were 41 mathematics lesson plans developed in the Curriculum Analysis and Mathematics Instruction Planning course.

The participating pre-service teachers had completed their third semester in the 2023/2024 academic year, during which they took Curriculum Analysis and Mathematics Instruction Planning and Strategies and Innovations in Mathematics Instruction. These courses provided the basis for selecting the participants. Participation was voluntary, and students' decisions about whether to participate had no effect on their course grades. The authors kept participants' identities confidential.

Data analysis followed Miles and Huberman's (1994) framework of data reduction, data display, and conclusion drawing and verification. Each of the 41 lesson plans served as a unit of analysis. During data reduction, both authors independently reviewed each lesson plan using the same assessment sheet. They recorded the curriculum phase and the instructional models, methods, or approaches specified in each plan. They also assessed the organization of mathematical content, the alignment of learning objectives and activities, and the evidence of anticipated student thinking and planned instructional responses. **Table 1** presents the criteria used in the analysis.

Anticipated student thinking was coded as present when a lesson plan described a specific strategy students might use, a response they might give, or an error or difficulty that might arise during a

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mathematical task. A general statement such as "students may have difficulty answering the question" was insufficient. A planned instructional response was coded as present when the plan specified what the teacher would ask, show, or do in response to the anticipated student thinking. A statement such as "the teacher provides guidance" was insufficient unless the plan described what that guidance would involve.

To verify the analysis, the authors compared their independent assessments and discussed the ratings and observations for all lesson plans. Each author contributed additional observations and interpretations from their review. When ratings differed, both authors returned to the relevant sections of the original lesson plan and reconsidered the evidence in relation to the assessment criteria. These discussions informed the final classifications and interpretations. This process constituted investigator triangulation (Carter et al., 2014). The final classifications were summarized in frequency tables, with each lesson plan counted once per category. A lesson plan could contribute to more than one category when it specified multiple instructional models, methods, or approaches. The authors reviewed the documented responses from the whole-class discussion to identify difficulties that pre-service teachers reported when anticipating students' mathematical thinking and planning instructional responses. These responses were summarized separately from the lesson-plan ratings and used to contextualize the patterns observed in the plans.

Table 1. Criteria for Analyzing Lesson Plans and Hypothetical Learning Trajectories

Dimension	Indicator	Evidence or coding rule
Curriculum context	Curriculum phase	Record the phase specified in the lesson plan.
Pedagogical competence	Instructional model, method, or approach	Record each model, method, or approach specified in the lesson plan. More than one may be recorded for the same plan.
Pedagogical competence	Alignment with the instructional model	Assess whether the planned activities follow the steps of the stated instructional model. Record as *aligned* or *not aligned*.
Professional competence	Mathematical topic	Record the mathematical topic addressed in the lesson plan.
Professional competence	Alignment, completeness, and sequence of mathematical content	Examine whether the content corresponds to the curricular learning outcomes, covers the required material, and is presented in a logical order. Record the category used in the analysis.
HLT learning objectives	Measurable verbs and alignment with curricular learning outcomes	Apply the four-point scale. **4** = all objectives use measurable verbs and the content aligns with the curricular learning outcomes. **3** = some objectives lack measurable verbs, while the content aligns. **2** = some objectives lack measurable verbs and some content is misaligned. **1** = none of the objectives use measurable verbs and the content is misaligned.
HLT learning activities	Sequence of planned activities	Record *yes* when the planned activities follow a logical sequence consistent with the order of the mathematical content. Otherwise, record *no*.
HLT learning activities	Support for the learning objectives	Judge whether the planned HLT activities support the stated learning objectives. Record *yes* or *no*.
HLT learning activities	Alignment with the hierarchy of mathematical content	Judge whether the proposed learning trajectory follows the hierarchy of the mathematical content. Record *yes* or *no*.
HLT anticipated student thinking	Presence of specific anticipated student thinking	Record *present* when the plan identifies a specific strategy, response, error, or difficulty related to a mathematical task. Otherwise, record *absent*.

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Dimension	Indicator	Evidence or coding rule
Planned instructional response	Presence of a specific follow-up	Record *present* when the plan specifies what the teacher would ask, show, or do in response to the anticipated student thinking. Otherwise, record *absent*.
HLT completeness	Extent to which the HLT is specified	Record *complete*, *partial*, or *absent* using the decision rules applied in the study.

RESULTS

The analysis included 41 mathematics lesson plans developed by pre-service teachers. Some plans covered more than one class meeting and specified different instructional models across meetings. Each lesson plan was treated as a single unit of analysis, with all instructional models specified in the plan recorded. A logical sequence of learning activities was documented more often than specific anticipations of student thinking or planned instructional responses. The results first describe the curriculum phases and instructional models represented in the plans, including how the planned activities aligned with the stated models. The next section examines how mathematical content, learning objectives, and activities were organized. The final section describes how the plans documented anticipated student thinking and planned instructional responses.

Curriculum Phases and Instructional Models

The 41 lesson plans covered lower and upper secondary mathematics. In Indonesia's *Kurikulum Merdeka*, Phase D generally corresponds to Grades 7–9, Phase E to Grade 10, and Phase F to Grades 11–12. The label F+ denotes Advanced Mathematics within Phase F (Rumah Pendidikan, 2025; Setiyani et al., 2025). As shown in **Table 2**, Phase D accounted for the largest share of the lesson plans, with 15 of the 41 plans (36.6%).

Table 2. Distribution of Lesson Plans by Curriculum Phase

Phase	Target level	Lesson plans, *n* (%)
D	Grades 7–9, Mathematics	15 (36.6)
E	Grade 10, Mathematics	10 (24.4)
F	Grades 11–12, Mathematics	6 (14.6)
F+	Grades 11–12, Advanced Mathematics	10 (24.4)
Total		**41 (100.0)**

Phase D was the largest individual category, while Phases E, F, and F+ together accounted for 26 of the 41 lesson plans. The distribution therefore shows that the largest single group of plans targeted lower secondary mathematics, while nearly two-thirds of the plans targeted upper secondary mathematics. At the upper secondary level, the corpus included plans for both Mathematics and Advanced Mathematics, providing a range of curricular settings for examining the instructional models and activities specified in the plans.

As shown in **Table 3**, problem-based learning was the most frequently recorded instructional model, followed by cooperative learning and discovery learning. The 41 lesson plans yielded 63 entries because some plans covered multiple class meetings and specified different instructional models across those meetings.

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Table 3. Frequency of Instructional Models and Approaches Recorded in the Lesson Plans

Instructional model or approach	Frequency
Problem-based learning	25
Cooperative learning	21
Discovery learning	15
Conventional instruction	1
Realistic Mathematics Education	1
Total recorded entries	**63**

The recorded model choices were concentrated in problem-based learning, cooperative learning, and discovery learning, which together accounted for 61 of the 63 entries. Some plans specified more than one model. For example, one plan listed both problem-based learning and cooperative learning. A single plan could therefore contribute to several categories, particularly when it covered multiple class meetings. This pattern makes the planned activities important for interpreting the model choices. The activity sequences show how the pre-service teachers translated the models they named into instructions and tasks.

As shown in **Table 4**, the planned activities aligned with the steps of the stated instructional models in 36 of the 41 lesson plans (87.8%). In the five plans classified as not aligned, the activities were described too broadly or did not clearly correspond to the steps of the stated model.

Table 4. Alignment of Planned Activities with the Stated Instructional Models

Alignment	Lesson plans, *n* (%)
Aligned	36 (87.8)
Not aligned	5 (12.2)
Total	**41 (100.0)**

The alignment analysis showed how consistently the stated models were reflected in the planned activities. In 36 of the 41 lesson plans, the activities followed the steps of the models identified in the plans. The five plans classified as not aligned illustrate where the connection was less clear. For example, one plan identified problem-based learning as its model but opened the activity sequence with a presentation of the mathematical content. The assessors recorded this opening as a mismatch with the planned model. In other cases, the activities were described in terms too general to identify the steps of the stated model. These cases show two ways a mismatch appeared in the written plans. In some plans, the activity sequence differed from the expected steps. In others, the description provided too little detail to trace those steps.

Mathematical Content, Learning Objectives, and Activities

Table 5 summarizes the classifications of mathematical content across the 41 lesson plans. Most plans were classified as aligned with the curricular learning outcomes. The remaining two were classified under issues of content completeness or sequence.

Table 5. Alignment, Completeness, and Sequence of Mathematical Content

Recorded classification	Lesson plans, *n* (%)
Aligned with curricular learning outcomes	39 (95.1)
Incomplete content	1 (2.4)
Content out of sequence	1 (2.4)
Total	**41 (100.0)**

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The two exceptions point to different issues in content planning. In the plan classified as incomplete, the assessment notes indicated that the material on composite geometric transformations omitted some of the expected combinations. The sequencing category concerns how the selected content was ordered for instruction. These distinctions help explain what the assessors examined beyond the choice of a topic.

Table 6 presents the ratings for the formulation of learning objectives. The scale considered both the use of measurable verbs in the objectives and the alignment of the mathematical content with the curricular learning outcomes.

Table 6. Ratings of Learning Objective Formulation

Score	Lesson plans, *n* (%)
4	36 (87.8)
3	5 (12.2)
2	0 (0.0)
1	0 (0.0)
Total	**41 (100.0)**

The difference between scores 3 and 4 concerned how consistently measurable verbs were used. In the five plans that received a score of 3, the content aligned with the curricular learning outcomes, but some objectives lacked measurable verbs. A plan could therefore cover the intended mathematics while leaving some of the expected student actions underspecified.

Table 7 shows whether the planned learning activities followed a logical sequence. Most plans presented activities in an order that the assessors judged to be coherent.

Table 7. Sequence of Planned Learning Activities

Activities followed a logical sequence	Lesson plans, *n* (%)
Yes	38 (92.7)
No	3 (7.3)
Total	**41 (100.0)**

This criterion captured how the activities were ordered within the lesson plan. Two related judgments considered what that sequence was designed to accomplish. **Table 8** reports whether the planned activities were judged to support the stated learning objectives.

Table 8. Support of Planned HLT Activities for the Learning Objectives

Activities judged to support the learning objectives	Lesson plans, *n* (%)
Yes	21 (51.2)
No	20 (48.8)
Total	**41 (100.0)**

Table 9 reports whether the proposed learning trajectory followed the hierarchy of the mathematical content.

Table 9. Alignment of the HLT Sequence with the Hierarchy of Mathematical Content

HLT sequence judged to follow the content hierarchy	Lesson plans, *n* (%)
Yes	19 (46.3)
No	22 (53.7)
Total	**41 (100.0)**

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The three ratings capture different aspects of a learning sequence. A plan may specify a clear order for classroom activities while leaving the connection between those activities and the learning objectives unclear. Similarly, the order in which classroom activities are scheduled and the order in which mathematical ideas build on one another are distinct features of a plan. Across the 41 plans, a logical activity sequence was recorded more often than either support for the learning objectives or alignment with the hierarchy of mathematical content.

Anticipated Student Thinking and Planned Instructional Responses

Table 10 shows whether the lesson plans documented specific anticipations of students' mathematical thinking. Twenty-two of the 41 plans identified a possible strategy, response, error, or difficulty that could arise during a mathematical task. The presence of an anticipation varied in scope. In one plan, the assessment notes recorded only a single anticipated student response. In another, the notes identified a response only in the opening activity. These examples show that a plan could document how students might respond at one point in the lesson while leaving their possible responses to later tasks unspecified.

Table 10. Anticipated Student Thinking in the Lesson Plans

Anticipated student thinking	**Lesson plans, *n* (%)**
Identified	22 (53.7)
Not identified	19 (46.3)
Total	**41 (100.0)**

Figure 1 shows how often the plans that anticipated student thinking also specified an instructional response. Of the 22 plans with an anticipation, five (22.7%) described a concrete follow-up action. The other 17 (77.3%) left the teacher's response unspecified. The gap shows where the written planning often stopped. The plan identified what a student might think or do, while leaving open what the teacher would ask, show, or do in response.

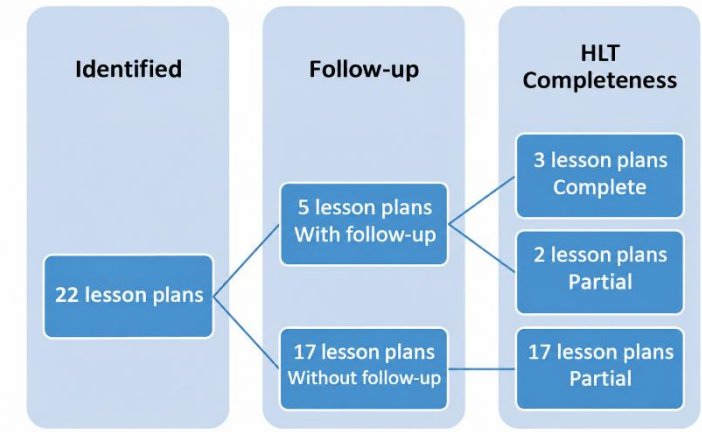

Figure 1. Planned Instructional Responses Among Lesson Plans with Anticipated Student Thinking

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Table 11 presents the classification of HLT completeness across the 41 lesson plans. The *partial* category was more common than the *complete* category, accounting for 19 plans compared with three classifieds as complete. The completeness ratings add a distinction to the presence of an anticipation. The counts in **Table 10** indicate whether a specific anticipation appeared in a plan. The predominance of the *partial* category over the *complete* category in **Table 11** indicates that documenting an anticipation at some point in a lesson was more common than developing a complete HLT.

Table 11. Completeness of the HLTs Documented in the Lesson Plans

HLT classification	**Lesson plans, *n* (%)**
Complete	3 (7.3)
Partial	19 (46.3)
No HLT identified	19 (46.3)
Total	**41 (100.0)**

Figures 2 and **Figure 3** illustrate how the lesson plans differed in their treatment of student thinking. **Figure 2** shows a lesson on correlation analysis that specifies individual reading and group discussion. The excerpt describes how students were expected to participate, but it does not identify a mathematical strategy, response, or difficulty that might arise during those activities. The classification of *no HLT identified* was based on the review of the full lesson plan. The excerpt illustrates the distinction between describing classroom participation and anticipating how students might think about the mathematics. In the excerpts classified as partial HLTs, the plans made an expected response explicit.

Kegiatan Inti Tahap *Think*	1. Guru membagikan bahan bacaan serta LKPD untuk setiap siswa mengenai materi analisis korelasi (terlampir) 2. Guru meminta peserta didik untuk membaca serta memahami materi analisi korelasi yang ada dibahan bacaan secara individu. 3. Peserta didik membaca dan memahami materi analisis korelasi secara individu. 4. Guru memberikan kesempatan kepada peserta didik yang ingin bertanya dan memberikan penjelasan terkait hal yang ditanyakan.	65 menit
Tahap *Talk*	1. Guru membagi 6 kelompok heterogen dengan masing masing kelompok terdiri dari 5 peserta didik dengan salah satu peserta didik yang memiliki kemampuan akademik yang baik untuk memimpin jalannya diskusi	

Figure 2. Planned Activities for a Lesson on Correlation Analysis

Figure 3a gives a worked solution to a task involving the Pythagorean theorem, and **Figure 3b** shows the expected mapping diagrams for two items. In both excerpts, the anticipation centres on the expected solution, leaving open how students might approach the task differently or where they might encounter difficulty. The *partial* classification reflects this limited scope. Each plan anticipates a specific mathematical response, and the excerpts leave alternative reasoning and follow-up actions unspecified.

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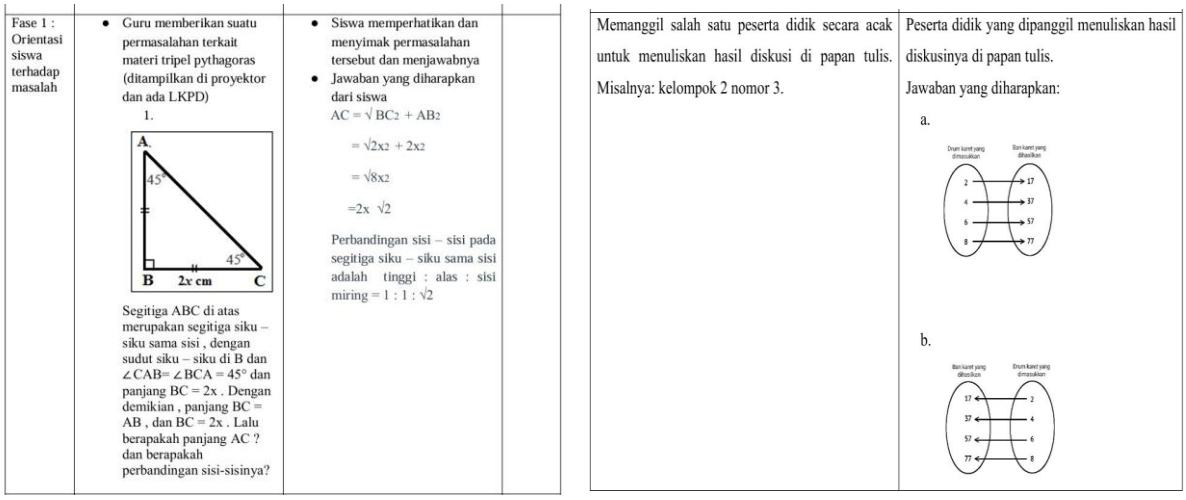

(a) (b)

Figure 3. Examples of Expected Student Responses in Partial HLTs

Note. Panel (a) shows an expected solution to a task involving the Pythagorean theorem. Panel (b) shows the expected mapping diagrams for two items.

Figure 4 illustrates how one lesson plan used students' answers to guide the next instructional decision. The plan specified that the teacher would proceed to vectors if students answered the prerequisite questions correctly and briefly revisit prerequisite content if their answers were inaccurate. The excerpt makes the two instructional options explicit. It was coded as a planned follow-up because the plan linked the quality of students' answers to what the teacher would do next. The review remains broad, as the plan lists several prerequisite topics without linking a particular error to a targeted explanation.

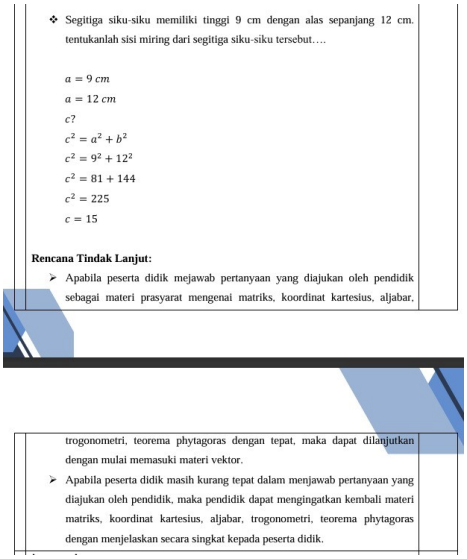

Figure 4. Planned Follow-Up Based on Students' Responses to Prerequisite Questions

During the whole-class discussion, pre-service teachers reported difficulty imagining how students might respond to a planned mathematical task. This uncertainty made it hard to plan a follow-up. To decide what to ask or show next, they first had to anticipate a specific response that the task might elicit. Some also explained that they expected to address difficulties as they arose in class. They preferred to decide how to respond after seeing how students approached the task and therefore saw little need to specify a follow-up in advance. This view offers one possible explanation for why few lesson plans documented a specific instructional response.

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DISCUSSION

The lesson plans showed two distinct aspects of planning. The pre-service teachers generally organized classroom activities in a logical order and specified how those activities would follow the selected instructional model. The links between tasks, learning objectives, and anticipated student thinking were less consistently documented. The discussion considers how these findings relate to the first research question on lesson design and the second on anticipating and responding to student thinking.

Instructional Models and the Organization of Learning Activities

The analysis of instructional models distinguished between naming a model and specifying how it would guide classroom activity. Most plans made the steps of the stated model visible in the activity sequence. In the plans classified as misaligned, the difficulty took two forms. Some sequences departed from the stated model, while other descriptions were too general for the assessors to trace its steps. This distinction matters because selecting an instructional approach and preparing to use it involve different planning decisions. Hammer and Ufer (2023) make a related distinction between selecting a mathematical task and preparing its implementation. In the present study, the activity sequence provided the evidence for how the selected approach was intended to work in the lesson.

A logical order of classroom activities was recorded more often than alignment with the hierarchy of mathematical content or support for the stated learning objectives. From an HLT perspective, an activity sequence also needs a mathematical rationale. Each task should have a discernible role in helping students develop the understanding specified in the learning goal (Simon & Tzur, 2004). Gulkilik (2024) found a related pattern in plans for teaching with multiple representations. The pre-service teachers included several forms of representation, but rarely planned prompts that would help students articulate the mathematical features shared across them. In the present study, the difference between the ratings for activity order and content hierarchy suggests that a clear sequence of classroom actions can coexist with a less explicit progression of mathematical ideas.

Anticipating Student Thinking and Planning a Response

Specific anticipations appeared more often than specific instructional responses. This difference points to a planning decision that goes beyond listing a possible answer or error. The pre-service teacher also has to decide what a response might reveal about the student's reasoning and what question, representation, or task could help the student develop that reasoning. Fiangga et al. (2021) reported a similar difficulty in an Indonesian Realistic Mathematics Learning course, where pre-service teachers struggled to use anticipated thinking and misconceptions to develop teaching strategies. The lesson plans examined here show a related pattern in a curriculum and instructional planning course.

The whole-class discussion offered one possible explanation for this pattern. Some pre-service teachers expected to address difficulties as they arose because they would have a clearer basis for responding once they saw students' actual work. That reasoning recognizes the uncertainty of classroom interactions. Stein et al. (2008) give anticipation and monitoring complementary roles in mathematical discussions. Anticipating before class gives teachers specific strategies or difficulties to look for. Monitoring during class gives them evidence for deciding which response fits the student's reasoning. From this perspective, a planned follow-up can be written as a provisional option that the teacher revises in light of what students actually do.

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For mathematics teacher education, one implication is to ask pre-service teachers to connect one task, one plausible student response, and one follow-up in the same plan. They could explain what the anticipated response reveals and what they would ask or show to move the student toward the learning goal. Rianasari and Guzon (2024) found that pre-service teachers working with realistic mathematics tasks valued students' reasoning but still faced difficulties guiding them to construct new mathematical knowledge during instruction. The present study locates a related difficulty in the written plans, where the link between anticipated reasoning and a specific instructional response was often left open.

CONCLUSION

The analysis of 41 mathematics lesson plans showed that pre-service teachers generally specified an instructional model and organized classroom activities in a logical sequence. The plans varied in how clearly those activities supported the stated learning objectives and followed the progression of mathematical ideas. Twenty-two plans documented specific anticipations of student thinking, and five specified a concrete instructional response. The whole-class discussion offered one possible explanation for this pattern. Some pre-service teachers expected to decide how to respond during the lesson, once they had seen how students approached the task.

In mathematics teacher education, a lesson-planning assignment could ask pre-service teachers to connect a mathematical task to a plausible student response and specify a follow-up that serves the learning objective. Making that connection explicit gives them a response to consider and adapt when they encounter students' actual reasoning.

The study examined lesson plans from a single cohort in one mathematics teacher education program. The findings concern the pre-service teachers' written plans and the difficulties they reported in the whole-class discussion. How they would respond to students during classroom instruction remains an open question. One of the authors also taught the course. Participation was voluntary, and students' decisions about whether to participate had no effect on their grades. The author's role as a course instructor may still have influenced how openly students discussed their planning difficulties in front of the class.

DECLARATIONS

Author Contributions: NFW and IR contributed equally to conceptualization, methodology, formal analysis, validation, manuscript drafting, and manuscript review and editing. NFW collected the research data.

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Ethics Approval and Consent: Participation was voluntary, and students were free to decline participation without any effect on their course grades. Participants' identities were kept confidential in the analysis and reporting of the findings.

Data Availability: The original lesson plans and assessment records are subject to confidentiality restrictions because they contain information about the participating students. Access to anonymized materials is subject to participant consent and applicable institutional requirements.

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Conflict of Interest: One author also served as an instructor for the participating cohort. This relationship is disclosed as a potential source of bias in the assessment and interpretation of the lesson plans.

Use of Generative AI: The authors used ChatGPT (OpenAI) to assist with literature searching, brainstorming idea and English language editing. ChatGPT's image-generation tool was also used to revise the text labels and presentation of the lesson-plan flowchart. The authors reviewed and edited the assisted content and take responsibility for the accuracy and integrity of the manuscript, including its references and figures.

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